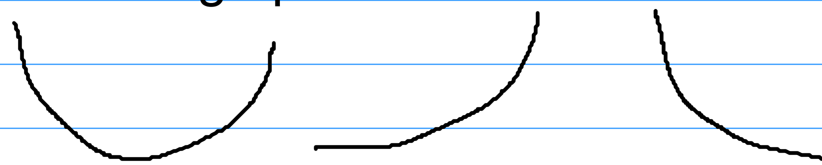
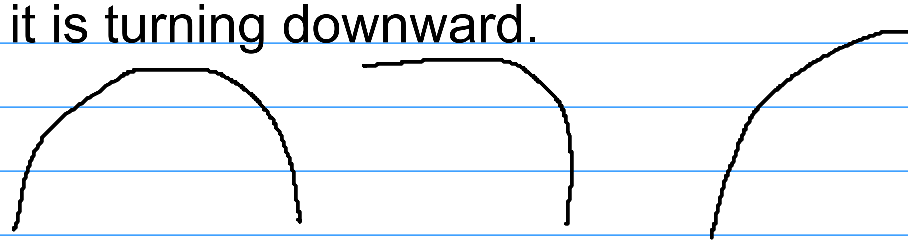
2.2 Using Second Derivatives: Maximums and Minimums

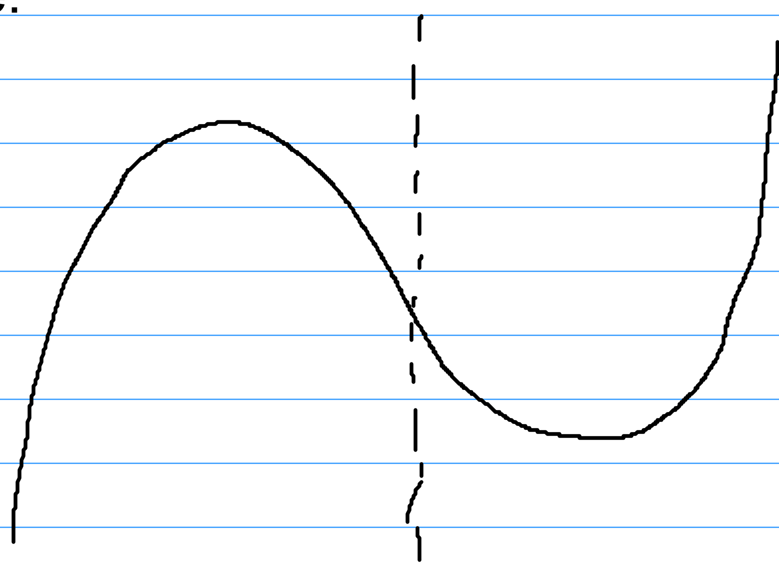
A function is concave up on an interval
if it is turning upward.



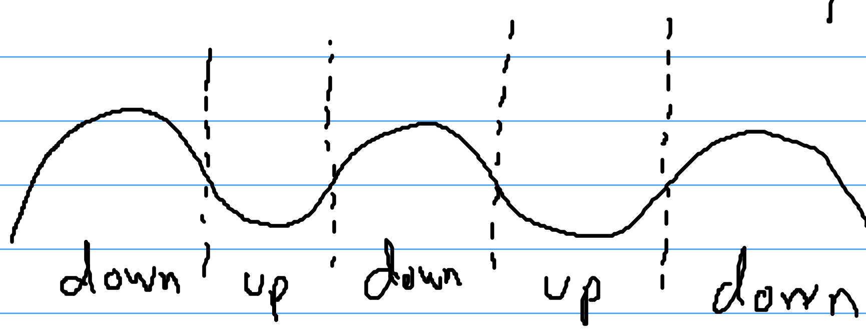
A function is concave down on an interval
if it is turning downward.



Example:



Concave down Concave up



On an interval:

f(x) is concave up

$\Longleftrightarrow$

the slopes of the tangent lines are increasing

$\Longleftrightarrow$

$f'(x)$ is increasing

$\Longleftrightarrow$

the derivative of $f'(x)$ is positive

$\Longleftrightarrow$

$f''(x) > 0$

The second derivative exactly describes the concavity of the function.

On an interval:

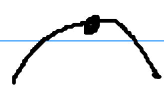
$f''(x) > 0 \iff f(x)$ is concave up

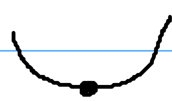
$f''(x) < 0 \iff f(x)$ is concave down

This leads to another way to determine if
a critical point ($f'(x)=0$ is a maximum or minimum.

<u>Second Derivative Test</u>

After finding critical points.

Max  $\iff f''(x) < 0$

Min  $\iff f''(x) > 0$

If $f''(x) = 0$, use First Derivative Test

Examples: (Using the same examples as in Section 2.1)

Find the local max's and min's of $f(x) = 2x^3 - 9x^2$

Critical Points:

$$f'(x) = 6x^2 - 18x$$
$$f'(x) = 0$$
$$6x^2 - 18x = 0$$
$$6x(x-3) = 0$$
$$x = 0,\ 3$$

$$f''(x) = 12x - 18$$

$x=0$: $f''(0) = 12(0) - 18 = -18 < 0$

maximum

$x=3$: $f''(3) = 12(3) - 18 = 36 - 18 > 0$

minimum

pts $x = 0$ $y = f(0) = 2(0)^3 - 9(0)^2 = 0$ $(0,0)$

$x = 3$ $y = f(3) = 2(3)^3 - 9(3)^2 = -27$ $(3,-27)$

Find the local max's and min's of $f(x) = x^4 + 4x^3$

Critical Points:

$$f'(x) = 4x^3 + 12x^2$$
$$f'(x) = 0$$
$$4x^3 + 12x^2 = 0$$
$$4x^2(x+3) = 0$$
$$x = 0 \qquad x = -3$$

$$f''(x) = 12x^2 + 24x$$

$x = -3$: $f''(-3) = 12(-3)^2 + 24(-3) = 108 - 72 > 0$ $\cup$

minimum

$x = 0$: $f''(0) = 12(0)^2 + 24(0) = 0 + 0 = 0$

Use First Derivative Test