2.1 Using First Derivatives: Max's and Min's
(More Examples)

$f(x) = x^4 + 4x^3$

Critical Points: $f'(x) = 0$

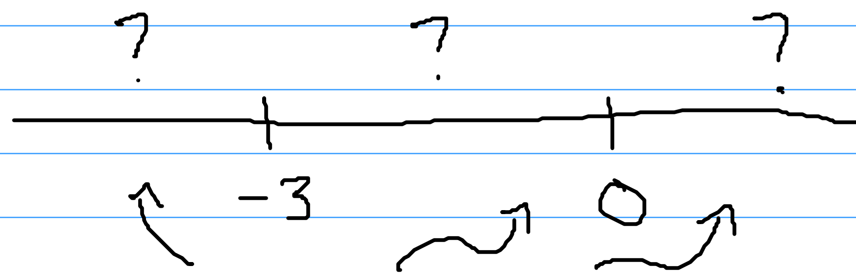
$$f'(x) = 4x^3 + 12x^2$$

$$4x^3 + 12x^2 = 0$$

$$4x^2(x+3) = 0$$

$4x^2 = 0$ $\qquad$ $x + 3 = 0$

$x = 0$ $\qquad$ $x = -3$



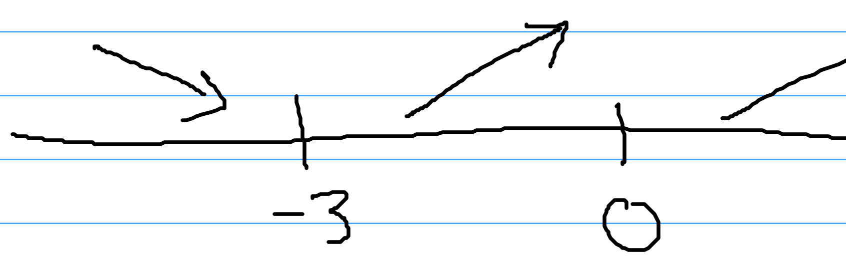
Test Points: -4, -1, 1

Inc / Dec : $f'+$ / $f'-$

$x = -4$: $f'(-4) = 4(-4)^2(-4+3) = 4 \cdot 16 \cdot (-1) < 0$ dec

$x = -1$: $f'(-1) = 4(-1)^3 + 12(-1)^2 = -4 + 12 > 0$ inc

$x = 1$: $f'(1) = 4(1)^3 + 12(1)^2 = 4 + 12 > 0$ inc



Local Minimum @ $x = -3$

Neither @ $x = 0$

$f(x) = \sqrt{x^2 + 2x + 4}$

Critical Points $\quad f' = 0$

$f(x) = (x^2 + 2x + 4)^{1/2}$

need Chain Rule

$f'(x) = \frac{1}{2}(x^2 + 2x + 4)^{-1/2}(2x + 2)$

$= (x^2 + 2x + 4)^{-1/2}(x + 1)$

$= \dfrac{x+1}{\sqrt{x^2 + 2x + 4}} = 0$

Numerator $= 0$

$x + 1 = 0$

$x = -1$

? ?

-1

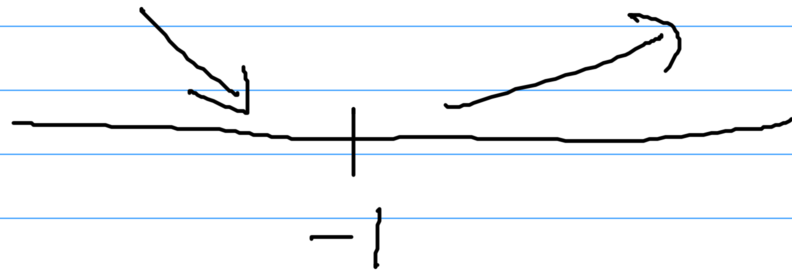
Test Points: -2, 0

Inc/Dec

$x = -2$: $\dfrac{-2+1}{\sqrt{(-2)^2 + 2(-2) + 4}} = \dfrac{-1}{\sqrt{4 - 4 + 4}} = \dfrac{-1}{\sqrt{4}} = \dfrac{-1}{2}$

< 0 dec ↘

$x = 0$: $\dfrac{0+1}{\sqrt{0^2+2(0)+4}} = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2} > 0$ inc $\nearrow$



Minimum @ $x = -1$