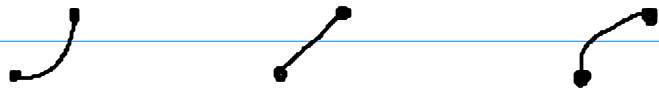
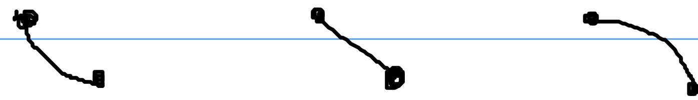
2.1 Using First Derivatives: Max's and Min's

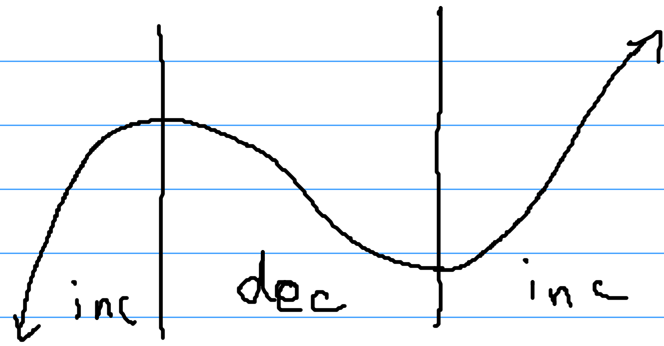
Def A function is increasing on an interval if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$



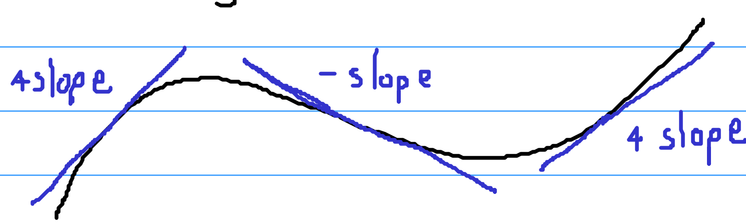
A function is decreasing if
if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$



Ex



The slope of the tangent line at each point tells us if the function is increasing or decreasing.

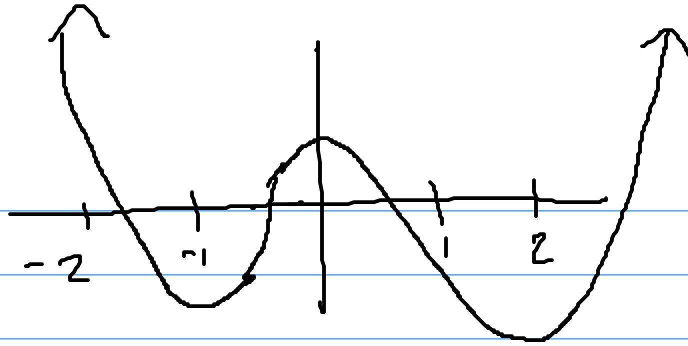


On an interval:

$f'(x) > 0 \iff f(x)$ is increasing ↗

$f'(x) < 0 \iff f(x)$ is decreasing ↘

Ex



$f'(x) > 0$ on $(-1, 0) \cup (2, \infty)$

$f'(x) < 0$ on $(-\infty, -1) \cup (0, 2)$

Def A <u>critical value</u> of $f(x)$ is $x = c$
where $f'(c) = 0$
or $f'(c)$ does not exist

Ex Find the critical points of $f(x) = x^2 - 6x + 5$

$$f'(x) = 2x - 6$$

want $f'(x) = 0$

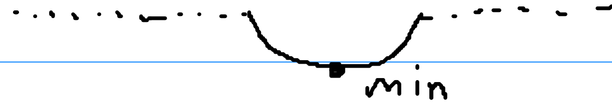
$$2x - 6 = 0$$

$$x = 3$$

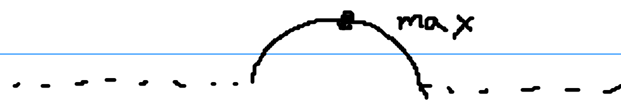
Graphically if $f'(c) = 0$ then
the slope of the tangent line is 0 at that pt
So the tangent line is horizontal.

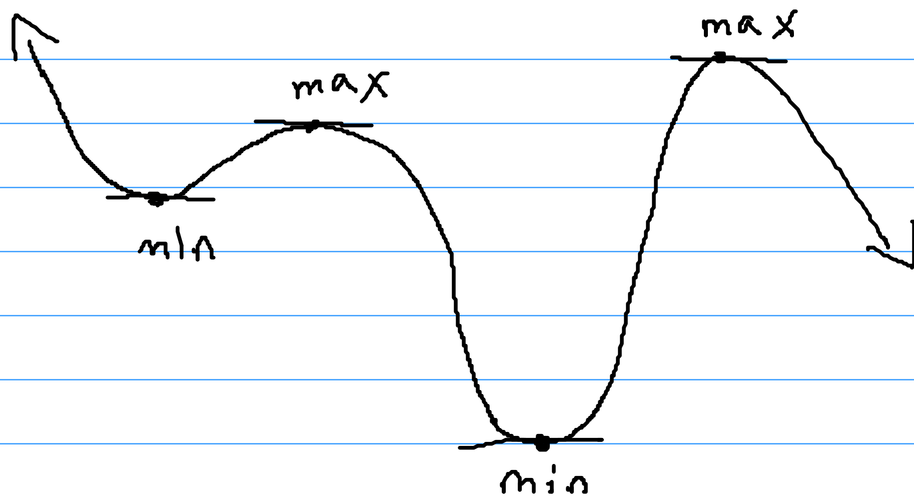
Critical pts on graphs:
- tangent lines are horizontal
- corners 

Def $x = c$ is a relative (or local) minimum if $f(c) \le f(x)$ in a small interval around $x = c$



Def $x = c$ is a relative (or local) maximum if $f(c) \geq f(x)$ in a small interval around $x = c$.



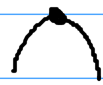


Big result: Local max's and min's occur at critical values!

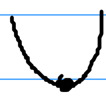
Once we have the critical values, how do we know which are max's and which are mins?

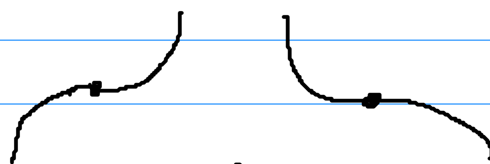
First Derivative Test

After finding the critical values ($f'=0$)

Max  : left of c.v. $\nearrow$ inc, $f'>0$
right of c.v. $\searrow$ dec, $f'<0$

$\nearrow$ inc $\cdot$ dec $\searrow$ $\qquad$ $\nearrow$ $f'>0$ $\cdot$ $f'<0$ $\searrow$

Min  : dec $\searrow$ $\cdot$ $\nearrow$ inc

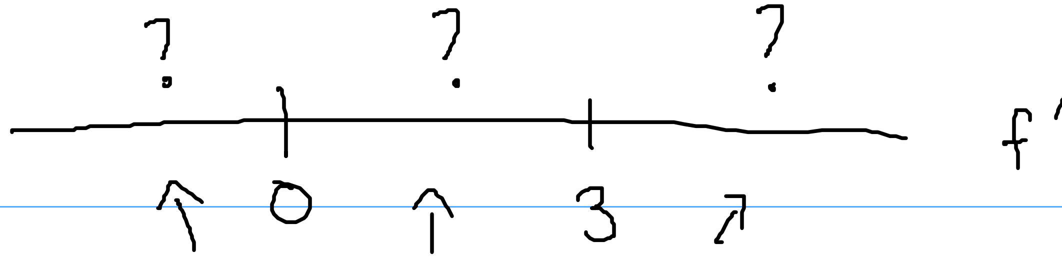
Neither  : $\nearrow$ inc $\cdot$ $\nearrow$ inc , dec $\searrow$ $\cdot$ dec $\searrow$

To find which critical values are max's and min's we'll construct a number line and determine increasing and decreasing regions with test points.

Ex $f(x) = 2x^3 - 9x^2 + 7$ Find max's and min's

Critical Points ($f'=0$)

$$f'(x) = 6x^2 - 18x$$
$$6x^2 - 18x = 0$$
$$6x(x-3) = 0$$
$$x = 0, 3$$



Test Points: -1, 1, 4

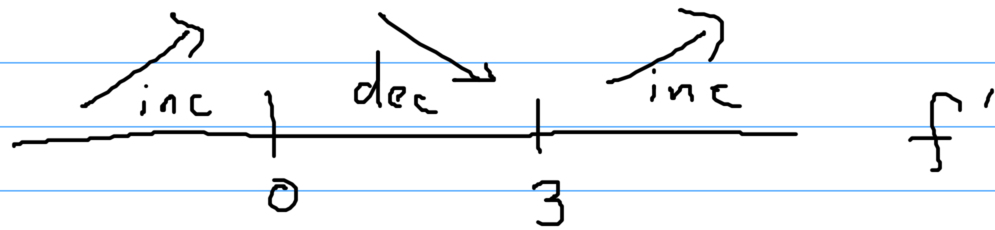
Test on $f'(x)$ for inc or dec

$x=-1$: $f'(-1) = 6(-1)^2 - 18(-1) = 6+18 > 0$ inc

$x=1$: $f'(1) = 6(1)^2 - 18(1) = 6 - 18 < 0$ dec

$x=4$: $f'(4) = 6(4)(4-3) = 24 > 0$ inc

factored version



0 is a max

3 is a min