1.6 Differentiation Techniques: Product Rule and Quotient Rule (part 1)

Recall $\frac{d}{dx} x^n = n x^{n-1}$ Power Rule

$y = x^5$ $y' = 5x^4$

$f(x) = 4x^{\frac{11}{8}}$ $f'(x) = 4 \cdot \frac{11}{8} x^{\frac{11}{8}-1} = \frac{11}{2} x^{3/8}$

$g(x) = x^7 + x^2$ $g'(x) = 7x^6 + 2x$

$h(x) = 8x^4 + 5$ $h'(x) = 32x^3 + 0$
$= 32x^3$

$y = x^3 + 4x^2 + 3x + 5 + x^{-1}$

$y' = 3x^2 + 8x + 3 - x^{-2}$

<u>Product Rule</u>

If $y = f(x) \cdot g(x)$

then $y' \neq f'(x) \cdot g'(x)$ WRONG

$y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Alternate: Let $u = f(x)$ $v = g(x)$

$(u \cdot v)' = u' \cdot v + u \cdot v'$

Ex $f(x) = \overset{u}{(3x+1)}\overset{v}{(x-4)}$ Find $f'(x)$

$u = 3x + 1$ $v = x - 4$

$u' = 3$ $v' = 1$

$$
\begin{aligned}
f'(x) &= u'v + uv' \\
&= 3(x-4) + (3x+1) \cdot 1 \\
&= 3x - 12 + 3x + 1 \\
&= 6x - 11
\end{aligned}
$$

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In this case, we can FOIL first and then take the derivative.

$f(x) = (3x+1)(x-4) = 3x^2 - 11x - 4$

$f'(x) = 6x - 11$

Ex $f(x) = 2x^3(7x+8)$ Find $f'(x)$

u v

$u = 2x^3$ $v = 7x + 8$

$u' = 6x^2$ $v' = 7$

$f'(x) = u'v + uv'$

$= 6x^2(7x+8) + (2x^3)(7)$

Ex $g(x) = (x^2+3)(x^2-2x+7)$

u v

$u = x^2 + 3$ $v = x^2 - 2x + 7$

$u' = 2x$ $v' = 2x - 2$

$g'(x) = u'v + uv'$

$= 2x(x^2-2x+7) + (x^2+3)(2x-2)$

Quotient Rule

If $y = \frac{f(x)}{g(x)}$

then $y' \neq \frac{f'(x)}{g'(x)}$

$y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Alternatively

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$