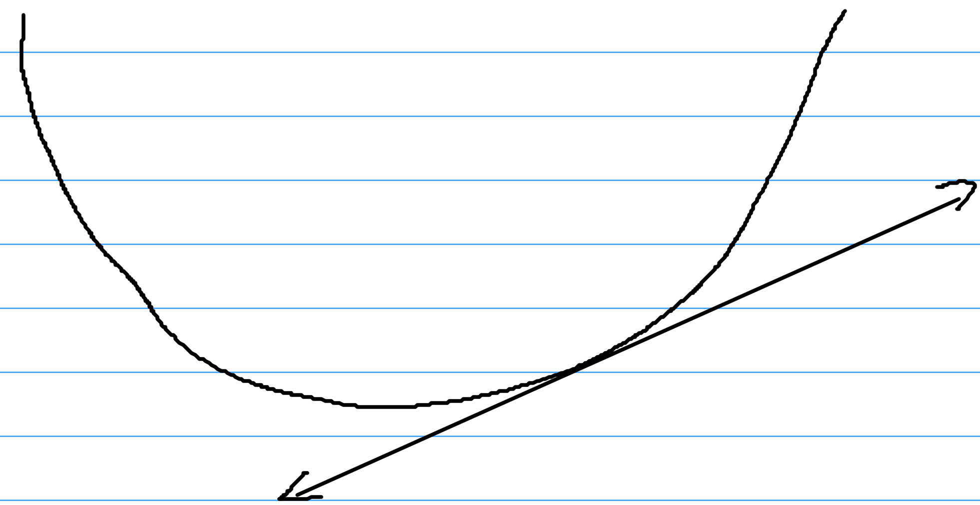
1.4 Differentiation Using Limits of Difference Quotients (part 2)



The slope of the tangent line to y=f(x) at x=a is

$$\boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Example $f(x) = x^2$

a) Find the slope of the tangent line at x=3. $a=3$

$$f(3+h) = (3+h)^2 = (3+h)(3+h)$$
$$= 9 + 6h + h^2$$
$$f(3+h) - f(3) = 9 + 6h + h^2 - 9$$
$$= 6h + h^2$$
$$\frac{f(3+h) - f(3)}{h} = \frac{6h + h^2}{h} = \frac{6h}{h} + \frac{h^2}{h}$$
$$= 6 + h$$
$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} (6+h)$$
$$= 6 + 0$$
$$= \boxed{6}$$

$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$
$$= \lim_{h \to 0} \frac{(3+h)^2 - (3)^2}{h}$$
$$= \lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h}$$
$$= \lim_{h \to 0} \frac{6h + h^2}{h}$$
$$= \lim_{h \to 0} 6 + h = 6 + 0 = \boxed{6}$$

b) Find the equation of the tangent line at $x=3$.

$m = 6$ $\quad$ pt = ?

$x = 3$ $\quad$ $y = f(3) = 3^2 = 9$

pt $(3, 9)$

$$y - 9 = 6(x-3)$$
$$y - 9 = 6x - 18$$
$$y = 6x - 9$$

Def

$f'(a)$ = slope of tangent line at $x=a$

so in the previous problem $\quad f'(3) = 6$

$f'(x)$ = a function that stores all slopes

At $x=a$, get $f'(a)$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$f'(x)$ is called the derivative of $f(x)$.

Example: Find the derivative of $f(x) = x^2 + 5x$

Difference Quot. $= 2x + h + 5$

$$f'(x) = \lim_{h \to 0} (2x + h + 5)$$
$$= 2x + 0 + 5$$
$$= 2x + 5$$

The slope to $f(x)$ at $x = 1$

$$f'(1) = 2(1) + 5$$
$$= 7$$

Steps to compute derivative

1) Find $f(x+h)$
2) Subtract $f(x)$
3) Divide by h
4) Take limit as $h \to 0$

Example $f(x) = 2x^2 - 3$ Find $f'(x)$

$$f(x+h) = 2(x+h)^2 - 3$$
$$= 2(x+h)(x+h) - 3$$
$$= 2(x^2 + 2xh + h^2) - 3$$
$$= 2x^2 + 4xh + 2h^2 - 3$$

$$f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 - 3 - (2x^2 - 3)$$
$$= \cancel{2x^2} + 4xh + 2h^2 \cancel{-3} \cancel{-2x^2} \cancel{+3}$$
$$= 4xh + 2h^2$$

$$\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2}{h} = \frac{4xh}{h} + \frac{2h^2}{h} = 4x + 2h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (4x + 2h) = 4x + 0 = \boxed{4x}$$

Find the tangent line at $x = 1$

$m = f'(1) = 4(1) = 4$

$x = 1 \quad y = f(1) = 2(1)^2 - 3$
$= -1$

$pt = (1, -1)$

$$y - (-1) = 4(x - 1)$$
$$y + 1 = 4x - 4$$
$$y = 4x - 5$$

Def A function with a derivative is differentiable.

What does a differentiable function look like?

Nice result: A differentiable function is continuous.

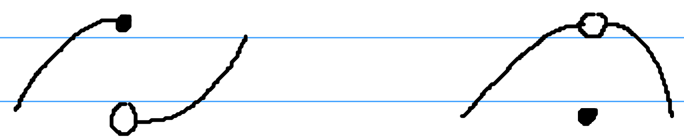
polynomials are differentiable

roots " where they exist

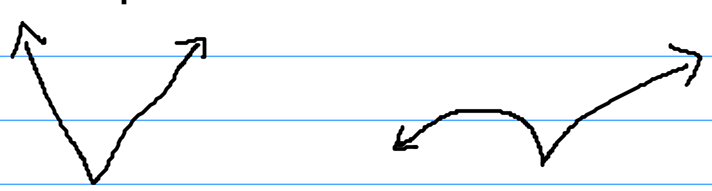
rational funcs ($\frac{1}{x}$) " " " "

A function does not have a derivative

- where it is not continuous



- at sharp corners



- at vertical tangents