1.2 Algebraic Limits and Continuity (part 2)

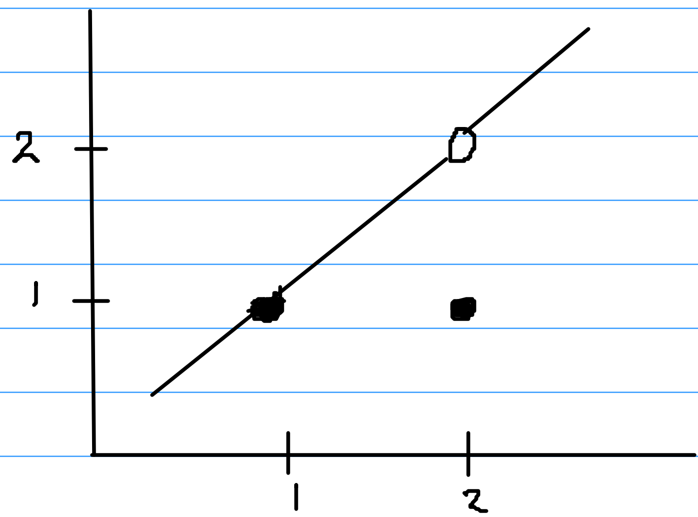
Continuity

$\lim_{x \to a} f(x)$ = what happens when "x is near a"

$f(a)$ = what happens when "x is at a"

Def f(x) is <u>continuous at x=a</u> if

$$\boxed{\lim_{x \to a} f(x) = f(a)}$$



$\lim_{x \to 1} f(x) = 1$

$f(1) = 1$

$\lim_{x \to 2} f(x) = 2$

$f(2) = 1$

f is continuous at x=1

f is <u>not</u> continuous at x=2

Def f(x) is discontinuous at x = a if
$$\lim_{x \to a} f(x) \neq f(a)$$

Def f(x) is continuous if
it is continuous at x=a for every a.
(continuous everywhere).

Result: polynomials (like $x^2 - 3x + 5$)
and
rational functions (like $\frac{x^2 - 4}{3x + 2}$)

are continuous where they exist.

Example. Where is $f(x) = x^2 + 7x + 2$ continuous?

Everywhere, since f(x) is a polynomial

Find $\lim_{x \to 1} f(x) = f(1)$ since continuous at $x=1$
$$= (1)^2 + 7(1) + 2$$
$$= 10$$

Example: Where is $f(x) = \dfrac{x^2}{x-4}$ continuous?

f(x) is not defined when the denominator = 0

$f(x)$ is contmuous except when $x=4$ $\quad x-4=0$, $x=4$

Find $\lim_{x \to 6} f(x)$

Since f(x) is continuous except at x = 4

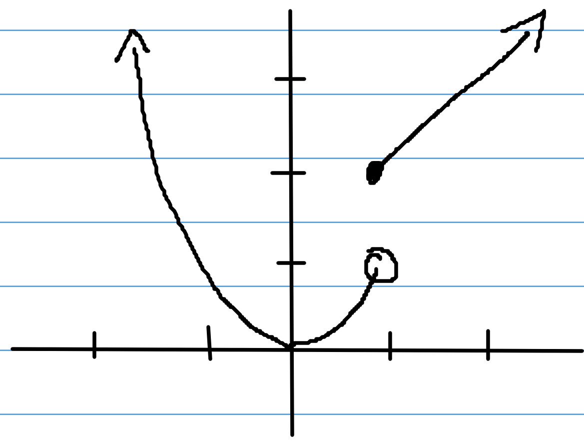
$f(x)$ is contmuous at $x=6$

$$\lim_{x\to 6} f(x) = f(6)$$

$$= \frac{6^2}{6-4} = \frac{36}{2} = 18$$

Continuity and piecewise functions

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ x+1 & \text{if } x \geq 1 \end{cases}$$



A discontinuity may exist on the boundary of two pieces.

We need to verify that: to show continuous at the boundary point

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

Example: Check f(x) for discontinuities.

$$f(x) = \begin{cases} x - 1 & x < 2 \\ 1 & 2 \leq x \leq 3 \\ x^2 - 5 & 3 < x \end{cases}$$

Possible discontinuities at $x = 2$ and $x = 3$

all equal

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x-1) = 2-1 = 1$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 1 = 1$$

$$f(2) = 1$$

f(x) is continuous at $x=2$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} 1 = 1$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x^2-5 = 3^2-5 = 4$$

$$f(3) = 1$$

f(x) is discontinuous at $x=3$

f(x) is continuous except at $x=3$