1.2 Algebraic Limits and Continuity (part 1)

Algebraic rules can be used to find new limits knowing other limits.

Limit Laws:

- $\lim_{x \to a} c \cdot f(x) = c \cdot \lim_{x \to a} f(x)$ c a constant
- $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
- $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
- $\lim_{x \to a} [f(x) \cdot g(x)] = [\lim_{x \to a} f(x)] \cdot [\lim_{x \to a} g(x)]$
- $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ except if denom = 0

Suppose we know that: $f(2) = 5$ $g(2) = 3$

$\lim_{x \to 2} f(x) = 4$ $\lim_{x \to 2} g(x) = 7$

Find $\lim_{x \to 2} [2f(x) - g(x)]$

$= \lim_{x \to 2} 2f(x) - \lim_{x \to 2} g(x)$

$= 2 \cdot \lim_{x \to 2} f(x) - \lim_{x \to 2} g(x)$

$= 2(4) - 7$

$= 8 - 7 = 1$

Find $\lim_{x \to 2} \dfrac{f(x)}{[g(x)]^2}$

$$= \frac{\lim_{x \to 2} f(x)}{\lim_{x \to 2} [g(x)]^2}$$ $g(x) \cdot g(x)$

$$= \frac{\lim_{x \to 2} f(x)}{\left(\lim_{x \to 2} g(x)\right)\left(\lim_{x \to 2} g(x)\right)}$$

$$= \frac{21}{7 \cdot 7} = \frac{4}{4\,9}$$

Educated guess for $\lim_{x \to 2} \sqrt{f(x)} = \sqrt{4} = 2$

Limit operations behave nicely with algebraic operations!

More Laws:

- $\lim_{x \to a} c = c$
- $\lim_{x \to a} x = a$
- $\lim_{x \to a} x^n = a^n$

Example: $\lim_{x \to 3} (x^2 + x - 4)$

$$= \lim_{x \to 3} x^2 + \lim_{x \to 3} x - \lim_{x \to 3} 4$$

$$= 3^2 + 3 - 4$$

$$= 9 + 3 - 4 = 8$$

fast way

Ex Find $\lim_{x \to 0} \frac{4x-3}{2x+7}$ (Fast Way)

$$= \frac{4(0)-3}{2(0)+7} = \frac{-3}{7}$$

Ex $\lim_{x \to 2} \frac{x^2-5x+6}{x^2-4}$

$$= \frac{4-10+6}{4-4} = \frac{0}{0}$$

$\frac{0}{0}$ form Try factoring.

(Recall that graphing and xy-chart are options)

$$\lim_{x \to 2} \frac{x^2-5x+6}{x^2-4} = \lim_{x \to 2} \frac{(x-3)(x-2)}{(x+2)(x-2)}$$

$$= \lim_{x \to 2} \frac{x-3}{x+2}$$

$$= \frac{2-3}{2+2} = \frac{-1}{4}$$