1.1 Limits: A Numerical and Graphical Approach

- Don't Forget Friday Videos
- MML Homework
- Demo written Homework on BB
- Lab Homework on BB Thursday

Limits are used to study the behavior of a function $f(x)$ near a given x value, but not at that x value.

Example $f(x) = \frac{x^2-9}{x-3}$ near $x=3$
(at $x=3$ $f(3)=\frac{0}{0}$ DNE)

Plug in x values closer and closer to 3 to see what f(x) is doing.

input goes to 3

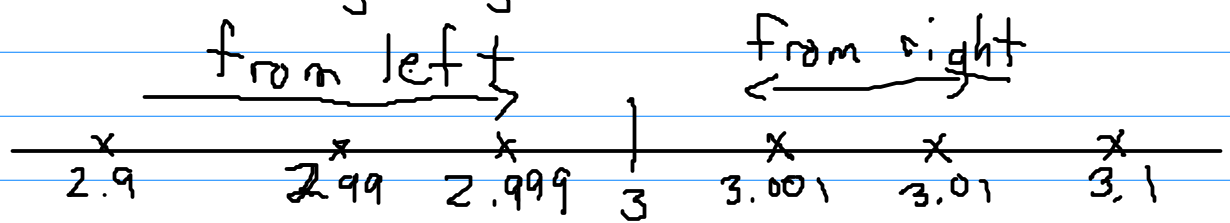
x	f(x)
2.9	5.9
2.99	5.99
2.999	5.999
2.9999	5.9999

output goes to 6

Notation $\lim\limits_{x \to a} f(x)$ represents the general limit of $f(x)$ as x approaches $x=a$.

So $\lim\limits_{x \to 3} \frac{x^2-9}{x-3} = 6$

One catch: the side that one approaches 3 on might give different values for f(x).



Example $f(x) = \frac{x}{|x|}$ x approaches 0

LEFT	
x	$f(x)$
$-.1$	$-.1/\lvert -.1 \rvert = -1$
$-.01$	-1
$-.001$	-1

RIGHT	
x	$f(x)$
$.1$	$.1/\lvert .1 \rvert = 1$
$.01$	1
$.001$	1

From the left and right give different values,

Def $\lim_{x \to a^-} f(x) =$ limit on left of $x = a$
think of $a -$ tiny number (.01 etc)
$\lim_{x \to a^+} f(x) =$ limit on right of $x = a$
think of $a +$ tiny number (.01 etc)

If left limit = right limit
then generic limit exists and
$\lim_{x \to a} f(x) =$ that limit

If left limit $\neq$ right limit
$\lim_{x \to a} f(x) = DNE$

So in the example $\lim_{x \to 0} \frac{x}{|x|} = DNE$

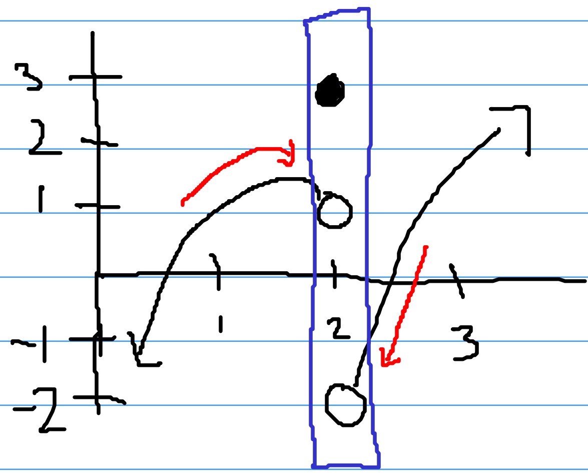
since $\lim_{x \to 0^-} \frac{x}{|x|} = -1$ $\lim_{x \to 0^+} \frac{x}{|x|} = 1$

$-1 \neq 1$

Graphical method for discovering limits:

To find the (left/right) limit at $x = a$,

- construct a wall at $x = a$
- follow the function into the wall from the (left/right) side
- the limit is the y-value where you hit the wall.

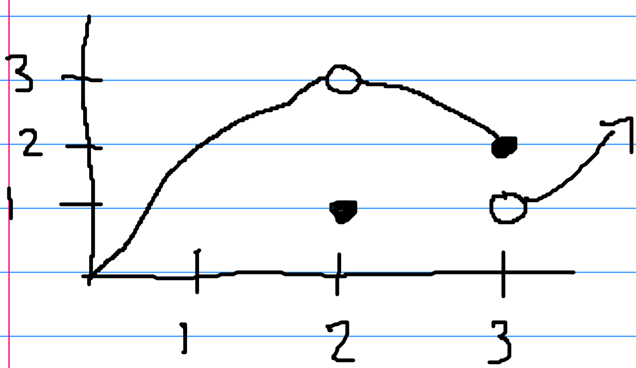


$$\lim_{x \to 2^-} f(x) = 1$$

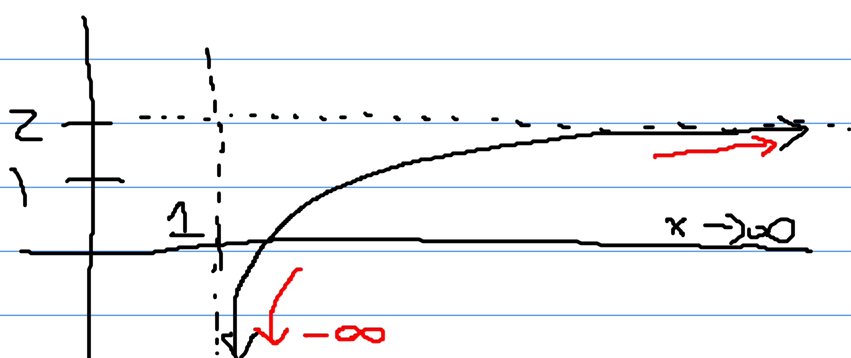
$$\lim_{x \to 2^+} f(x) = -2$$

$$\lim_{x \to 2} f(x) = DNE \quad 1 \neq -2$$

$$f(2) = 3$$



	Left	Right
$\lim_{x \to 3} f(x) = DNE$	2	$\neq$ 1
$\lim_{x \to 2} f(x) = 3$	3	= 3
$\lim_{x \to 1} f(x) = 2$	2	= 2



$$\lim_{x \to 1^+} f(x) = -\infty$$

$$\lim_{x \to \infty} f(x) = 2$$