R.4 Slope and Linear Functions (Point-Slope + Cost Analysis)

R.5 Nonlinear Functions and Models

Point-Slope Equation

$y - y_1 = m(x - x_1)$

where point = (x_1, y_1)

slope = m

Example: Find the line with slope m=2 continaing point (3,1)

$y - 1 = 2(x - 3)$

$y - 1 = 2x - 6$

$y = 2x - 5$

With two points, first find the slope.
Next, use a point and the slope to find the line.

Example: Find the line continaing points (3,2) and (5,0)

$x_1 \; y_1 \qquad x_2 \; y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{5-3} = \frac{-2}{2} = -1$$

$m = -1 \qquad pt = (3,2)$

$y - 2 = -1(x-3)$

$y - 2 = -x + 3$

$y = -x + 5$

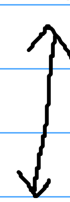
$m = -1 \qquad pt = (5,0)$

$y - 0 = -1(x-5)$

$y = -x + 5$

answers match

Special Lines:

	Horizontal	Vertical
Slope:	$m = 0$	m is undefined
Equation:	$y = \text{value}$	$x = \text{value}$
Picture:		

Application of Lines - Cost Analysis

$C(x) =$ Production Costs

$C(x) = mx + b$

where

$x =$ # of units

$m =$ marginal cost per unit

$b =$ fixed cost

Example: The cost of stamping one DVD is $0.20.
The machine to do it costs $100,000.

$m =$ \$0.20 $\qquad b =$ \$100,000

$$C(x) = 0.20x + 100{,}000$$

$R(x) =$ Revenue

$R(x) = px$

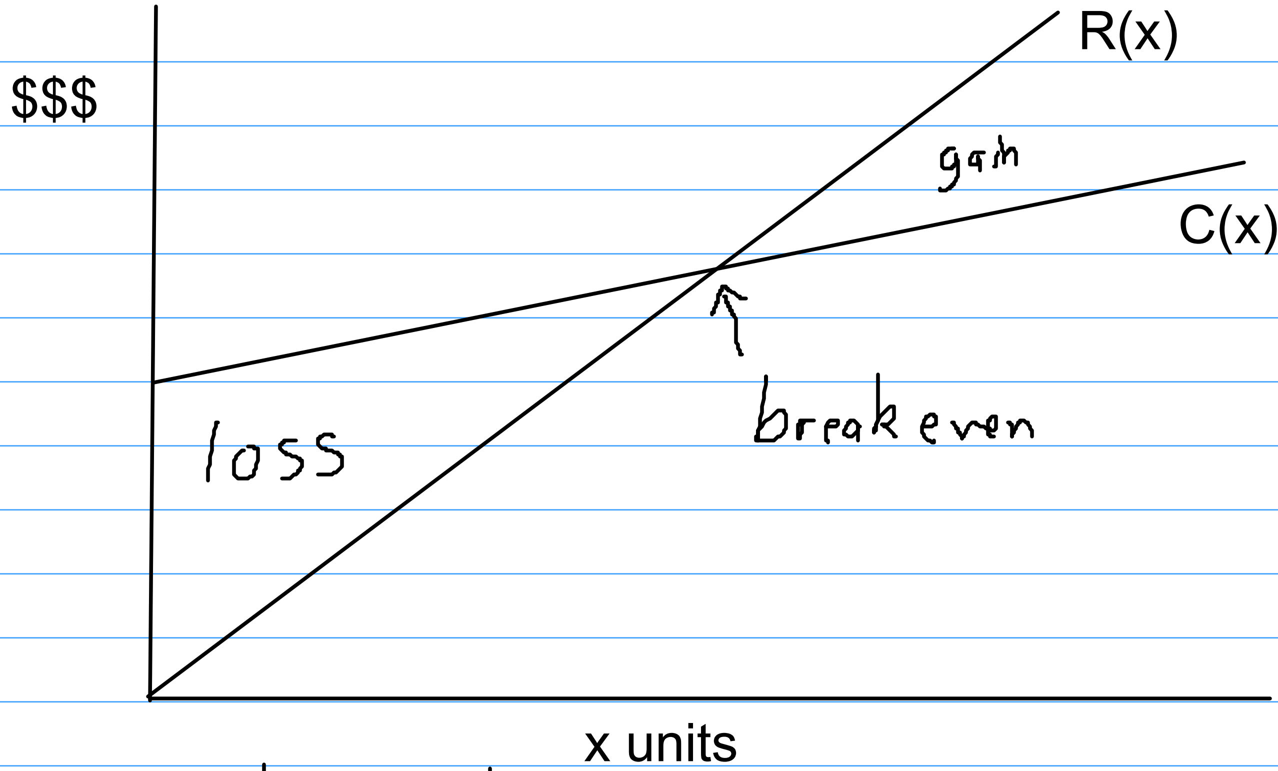
where

$x =$ # of unit

$p =$ price per unit

$P(x) =$ Profit

$$P(x) = R(x) - C(x)$$



breakeven when

$$R(x) = C(x)$$

Example: Red Dye #3 sells for $15 per gallon.
Materials to make it costs $10 per gallon,
and the mixer costs $1000 as a fixed cost.

Question: How many gallons need to sell to break even?

$m = 10 \qquad b = 1000 \qquad p = 15$

$C(x) = 10x + 1000 \qquad R(x) = 15x$

$$C(x) = R(x)$$

$$10x + 1000 = 15x$$

$$\frac{1000}{5} = \frac{5x}{5}$$

$$x = 200 \text{ units}$$

Break even cost/revenue $C(200) = R(200) = \$3{,}000$

R.5 Nonlinear Functions and Models

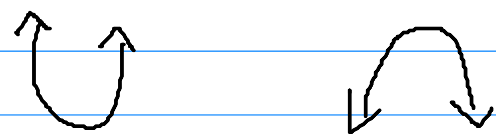
A linear function can be represented as

$$f(x) = ax + b \qquad (a \neq 0)$$

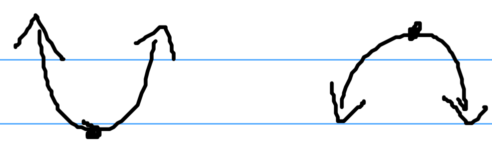
A quadratic function can be represented as

$$f(x) = ax^2 + bx + c \qquad (a \neq 0)$$

The graph of a quadratic function is a parabola



The maximum / minimum point of a parabola is called its vertex.

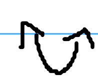


For $y = f(x) = ax^2 + bx + c$ the vertex occurs at

$$x = \frac{-b}{2a}$$

$$y = f(x) = f\left(\frac{-b}{2a}\right)$$

think

The graph opens up if $a > 0$  $y = x^2$

down if $a < 0$  $y = -x^2$

$a=1 \quad b=-2 \quad c=-8$

Example: Sketch $f(x) = x^2 - 2x - 8$

Vertex

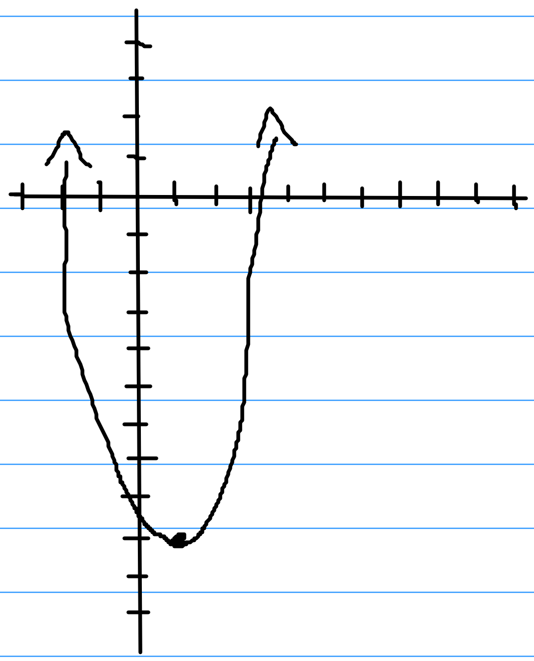
$x = -\frac{b}{2a} = -\frac{(-2)}{2(1)} = \frac{2}{2} = 1$

Opens up

$a > 0$

$y = f(1) = 1^2 - 2(1) - 8 = -9$

$(1, -9)$



The Quadratic Formula

Used to solve the equation:

$ax^2 + bx + c = 0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example: Find the zeros of $x^2 - 2x - 8$ $\quad a = 1 \quad b = -2 \quad c = -8$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 + 32}}{2}$$

$$= \frac{2 \pm \sqrt{36}}{2}$$

$$= \frac{2 \pm 6}{2}$$

$$x = \frac{2+6}{2} \text{ or } \frac{2-6}{2} \quad \to \quad x = 4 \text{ or } -2$$

Note: The online homework does not have the $\pm$ symbol.
Always enter in two answers, the + and - answers.

Recall how to simplify square roots.

Ex. $\sqrt{18} = \sqrt{2 \cdot 3 \cdot 3} = 3\sqrt{2}$

$\sqrt{28} = \sqrt{2 \cdot 2 \cdot 7} = 2\sqrt{7}$

$\sqrt{60} = \sqrt{2 \cdot 2 \cdot 3 \cdot 5} = 2\sqrt{15}$

Quadratic Formula and Graphs

$y = ax^2 + bx + c$

and $0 = ax^2 + bx + c$

So $y = 0$

These are the roots or x-intercepts of the graph.

In our example:

$y = x^2 - 2x - 8$

vertex = (1,-9)

zeros x = -2, -4