Math 1730 Test #2 Review

Chapter 1 (section 1.7, 1.8)

1. Find the derivative of each of the following:

a. $f(x)=3(x^2-5x-12)^4$ b. $f(x)=4\sqrt{6x^2-7x}$

c. $f(x)=\dfrac{-4}{(7-3x)^2}$ d. $f(x)=(3x-2)^4(1-x^5)^2$

2. Find the second derivative of each of the following:

a. $f(x)=7x^4-2x^2+9$ b. $f(x)=\dfrac{x-5}{2x+1}$ c. $f(x)=5\left(4x^2-7\right)^3$

3. Find $f^{(4)}(x)$ given that $f(x)=16x^{1/2}-4x^{3/2}$

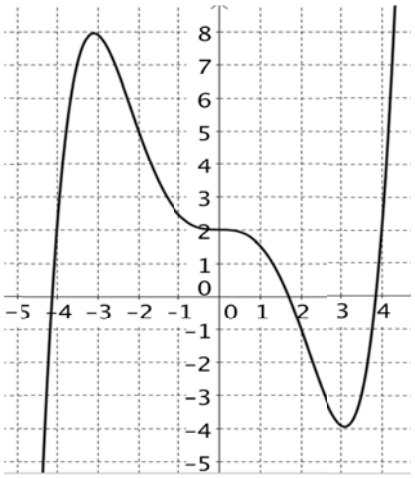
Chapter 2 (sections 2.1, 2.2, 2.3, 2.4, 2.5, 2.6)

4. Find the relative extreme points (maximums and minimums) of

$$f(x)=x^3+3x^2-24x+10.$$

5. Find the relative extrema (maximums and minimums) of $f(x)=-2x^3+1.$

6. Given the function *f(x)* graphed below, fill in the following:



a. *f* has a relative minimum when *x* = _____

b. *f* has a relative maximum when *x* = _____

c. *f* has a point of inflection when *x* = _____

d. *f* is increasing on the interval(s) _____

e. *f* is decreasing on the interval(s) _____

f. *f* is concave up on the interval(s) _____

g. *f* is concave down on the interval(s) _____

7. Find the absolute maximum and absolute minimum over the indicated interval, and indicate the *x*-values at which they occur:

$$f(x)=12+9x-3x^2-x^3 \text{ over } [0, 3]$$

8. Given $f(x)=x^3-2x^2-4x+3$ find the:

a. critical values
b. *x*-values of any relative extrema
c. *x*-values of any points of inflection
d. interval(s) in which *f* is increasing
e. interval(s) in which *f* is decreasing
f. interval(s) in which *f* is concave up
g. interval(s) in which *f* is concave down

9. Given $f(x) = x^7 - 7x^6 + 10$ find:

a. critical values
b. *x*-values of any relative extrema
c. *x*-values of any points of inflection
d. interval(s) in which *f* is increasing
e. interval(s) in which *f* is decreasing
f. interval(s) in which *f* is concave up
g. interval(s) in which *f* is concave down

10. Find the horizontal asymptotes (use limits) and vertical asymptotes for each of the following functions:

a. $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 9}$ b. $f(x) = \dfrac{-2x}{x^2 - 1}$

11. Find the following limits:

a. $\lim\limits_{x \to \infty} \dfrac{3x^2 - 5x + 1}{4 - x^2}$ b. $\lim\limits_{x \to \infty} \dfrac{2x^2 - 4x}{x + 5}$

12. A company decides that in order to sell *x* graphing calculators, the price per calculator must be $p = 150 - 0.75x$. It also determines that the total cost of producing *x* calculators is given by $C(x) = 3000 + 0.5x^2$

a. Find the total revenue, *R(x)*
b. Find the total profit, *P(x)*
c. How many calculators must the company produce and sell to maximize profit?
d. Find the maximum profit.
e. What price per calculator must be charged to maximize profit?

13. A car dealer expects to sell 800 cars a year. The cars cost $9000 each plus a fixed charge of $1000 per delivery. If it costs $1000 to store a car for a year, find the order size and the number of orders that minimize inventory costs.

14. A store can sell 20 bicycles per week at a price of $400 each. The manager estimates that for each $10 price reduction she can sell two more bicycles per week. The bicycles cost the store $200 each. Find the price that will maximize the manager's annual profit. Also state what her maximum profit will be.

15. A company finds that its monthly revenue, in dollars, from the sale of *x* leather bags is $R(x) = .006x^3 - 0.4x^2 + 130x$ and the cost is $C(x) = 40x + 100$. Currently, the company is selling 32 bags per month.

a. Find the monthly revenue when the company sells 32 bags.
b. Find the marginal revenue function.
c. Find the marginal revenue when 32 bags are sold.
d. Use the marginal revenue to estimate the total revenue when 33 bags are sold.
e. Find the marginal cost function.
f. Find the marginal cost when 32 bags are sold.
g. Find the marginal profit function.
h. Find the marginal profit when 32 bags are sold

Solutions

1a. $12(x^2-5x-12)^3(2x-5)$ 1b. $2(6x^2-7x)^{-\frac{1}{2}}(12x-7)$

1c. $-24(7-3x)^{-3}$ or $\dfrac{-24}{(7-3x)^3}$

1d. $(3x-2)^4\cdot 2(1-x^5)(-5x^4)+4(3x-2)^3\cdot 3(1-x^5)^2$
or $-10x^4(3x-2)^4(1-x^5)+12(3x-2)^3(1-x^5)^2$

2a. $84x^2-4$ 2b. $-44(2x+1)^{-3}$ **or** $\dfrac{-44}{(2x+1)^3}$

2c. $1920x^2(4x^2-7)+120(4x^2-7)^2$

3. $-15x^{-7/2}-\dfrac{9}{4}x^{-5/2}$

4. Relative minimum at (2, -18); relative maximum at (-4, 90)

5. No relative minimum or relative maximum

6a. 3 6b. -3 6c. 2, -2, 0

6d. $(-\infty, -3), (3, \infty)$ 6e. (-3, 3) 6f. $(-2, 0), (2, \infty)$

6g. $(-\infty, -2), (0, 2)$

7. Absolute maximum of 17 an $x = 1$, absolute minimum of – 15 at $x = 3$

8a. $x = -2/3$, $x = 2$

8b. Relative maximum at $x = -2/3$, relative minimum at $x = 2$

8c. point of inflection at $x = 2/3$

8d. $\left(-\infty, -\frac{2}{3}\right), (2, \infty)$

8e. $\left(-\frac{2}{3}, 2\right)$

8f. $\left(\frac{2}{3}, \infty\right)$

8g. $\left(-\infty, \frac{2}{3}\right)$

9a. $x = 0$, $x = 6$

9b. Relative maximum at $x = 0$, relative minimum at $x = 6$

9c. point of inflection at $x = 5$ 9d. $(-\infty, 0), (6, \infty)$

9e. $(0,6)$ 9f. $(5, \infty)$ 9g. $(-\infty, 5)$

10a. horizontal: $y = 1$, vertical: $x = 3$

10b. horizontal: $y = 0$, vertical: $x = 1$, $x = -1$

11a. -3 11b. ∞

12a. $R(x)=150x-0.75x^2$ 12b. $P(x)=-1.25x^2+150x-3000$

12c. 60 12d. \$1,500.00 12e. \$105.00

13. 40 cars per order, 20 orders per year

14. \$350 each, profit: \$4500.

15a. \$3947.01 15b. $R'(x) = 0.018x^2 - 0.8x + 130$

15c. \$122.83 15d. \$4069.84

15e. $C'(x) = 40$ 15f. \$40

15g. $P'(x) = 0.018x^2 - 0.8x + 90$ 15h. \$82.83