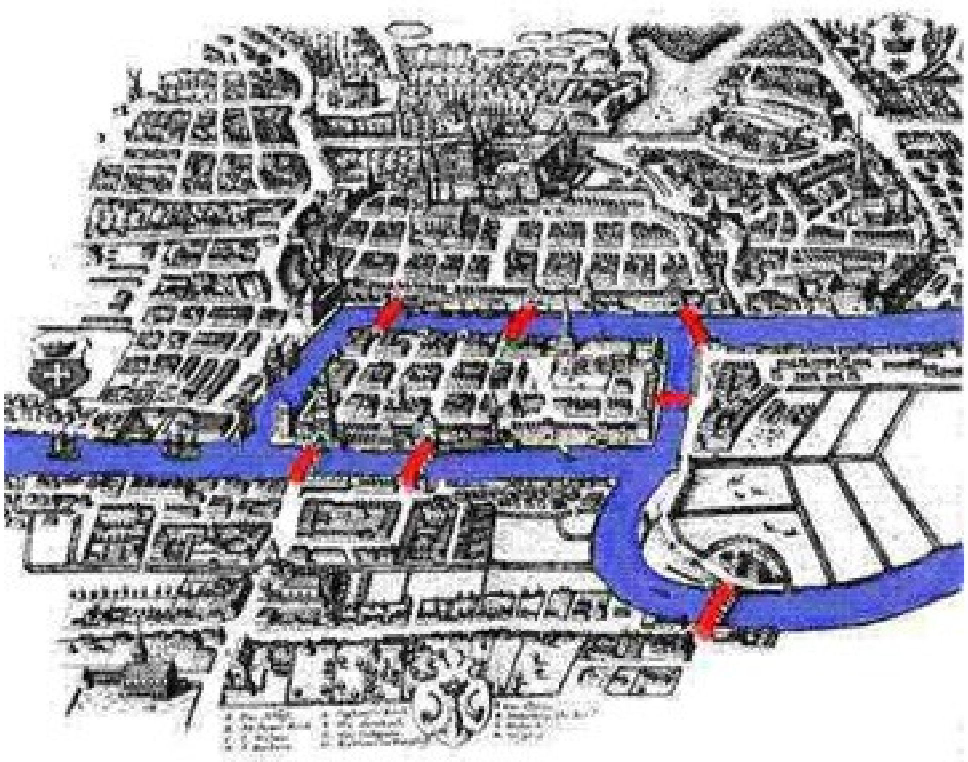
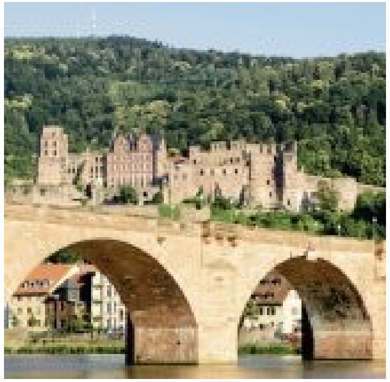
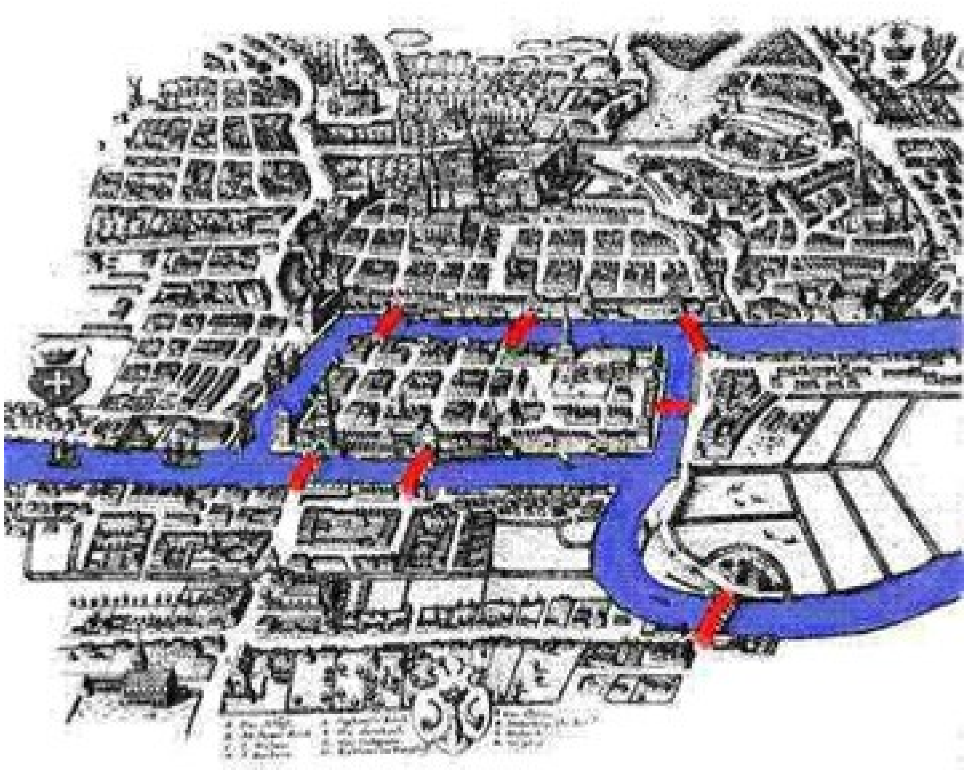
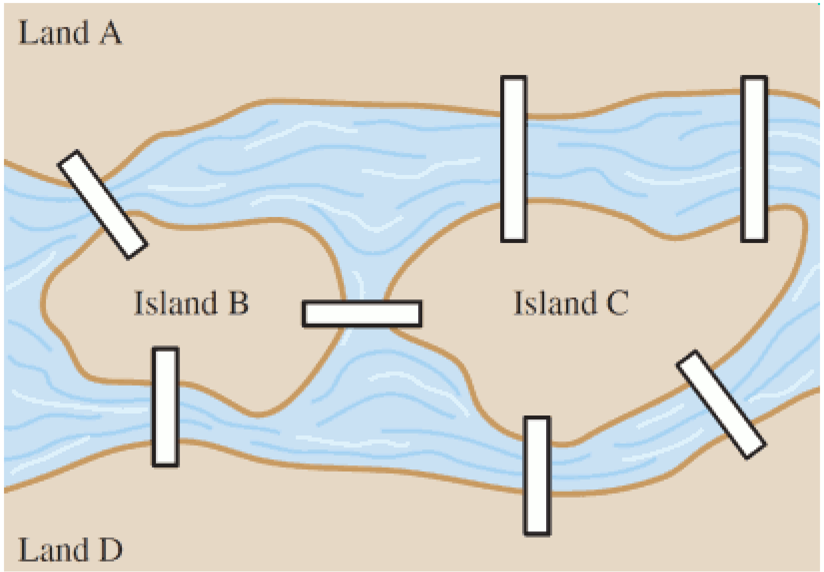
Koenigsberg Bridge Problem





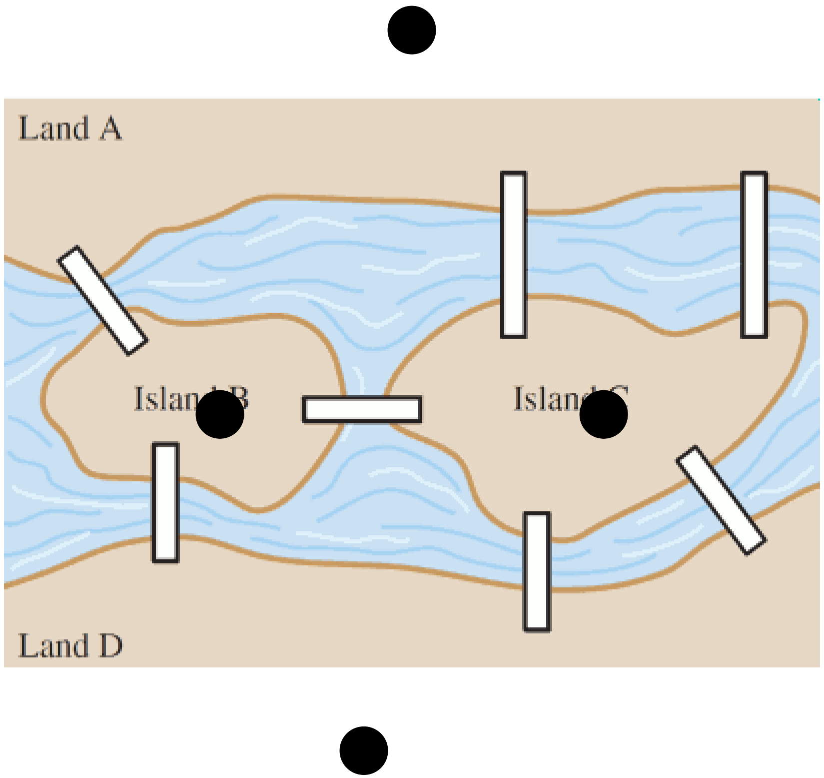




Can you find a way to cross all of the bridges, but do so only once?

Leonard Euler - Mathematician

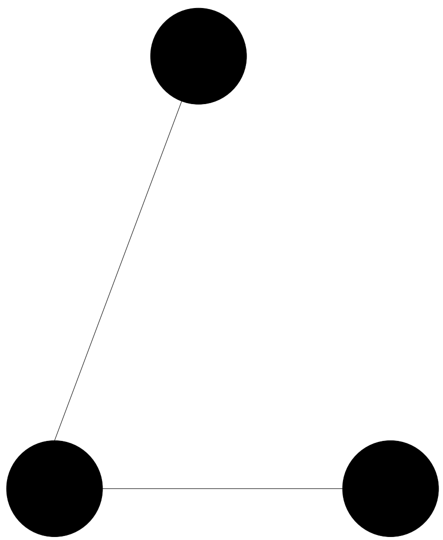
Found out about this problem and solved it in 1735.

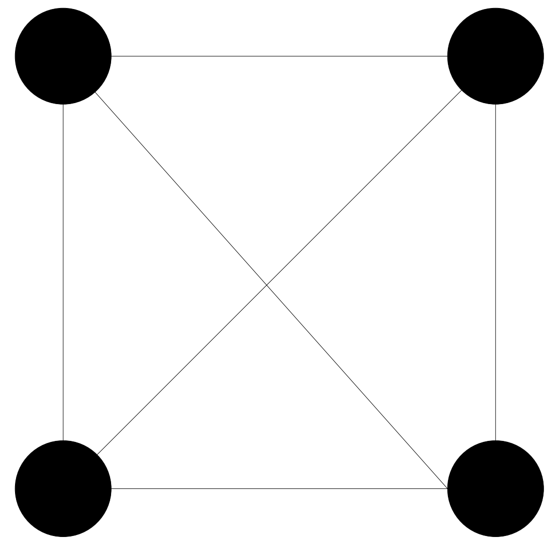


Special parts:

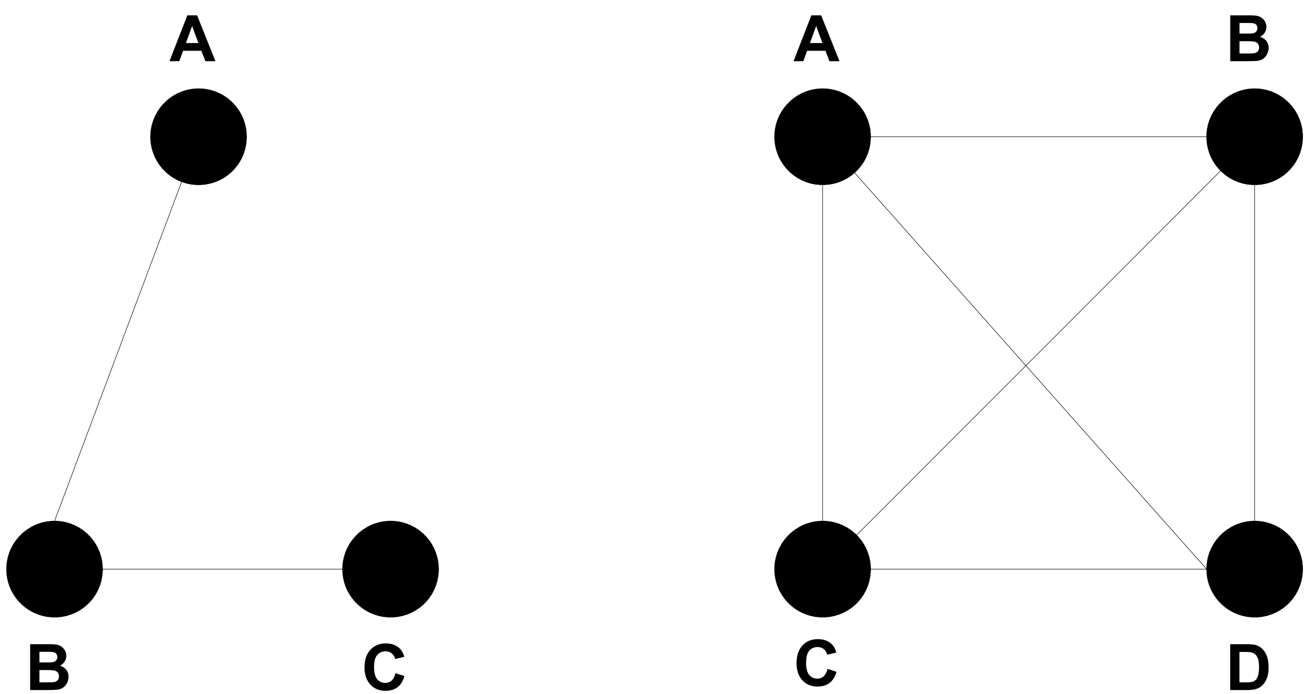
- Islands/Land
- Bridges

A **graph** is an object consisting of only **verticies** (points) and **edges** connecting them (lines)

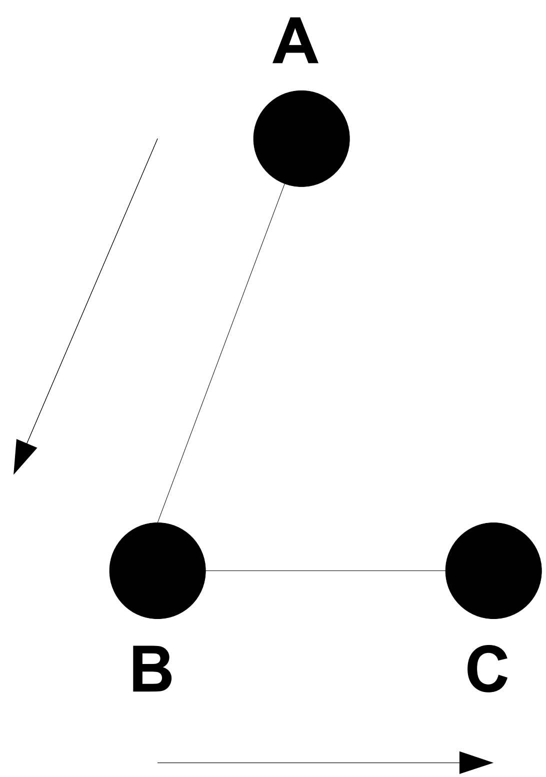




Verticies can be labeled.



A **path** in a graph is a traversal of verticies, across edges, in which no edge is used twice. (i.e. vertex to vertex without picking up the pencil)



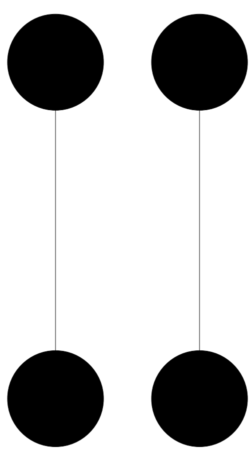
Here is the path
A → B → C

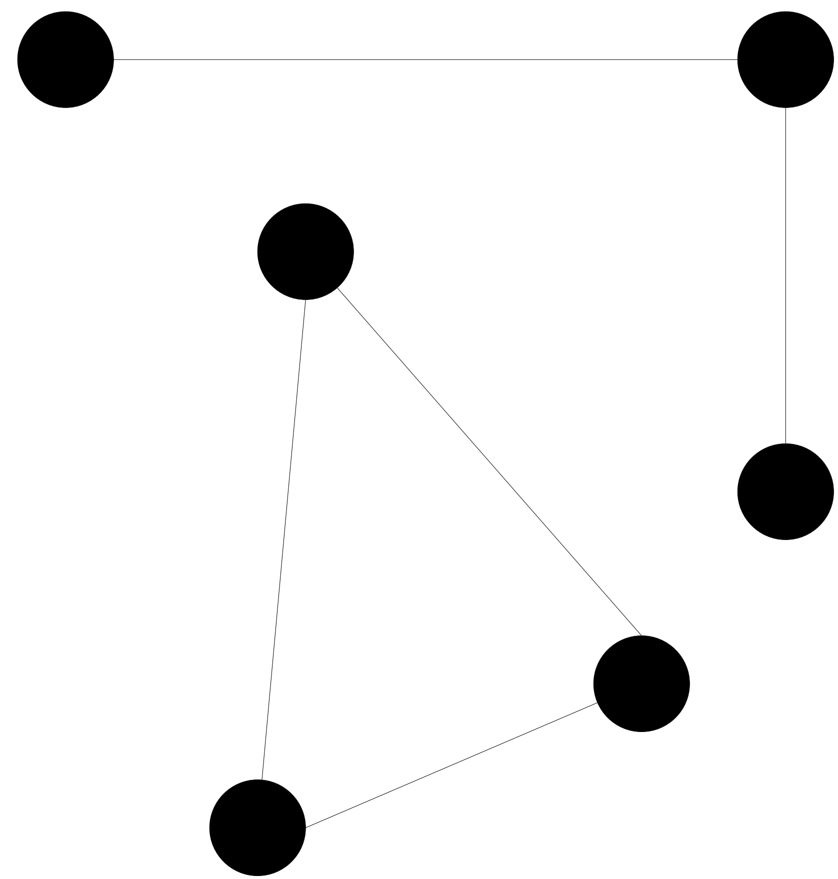
Real World Examples:

Kevin Bacon Numbers

Street Maps

What can you say about these 2 graphs?





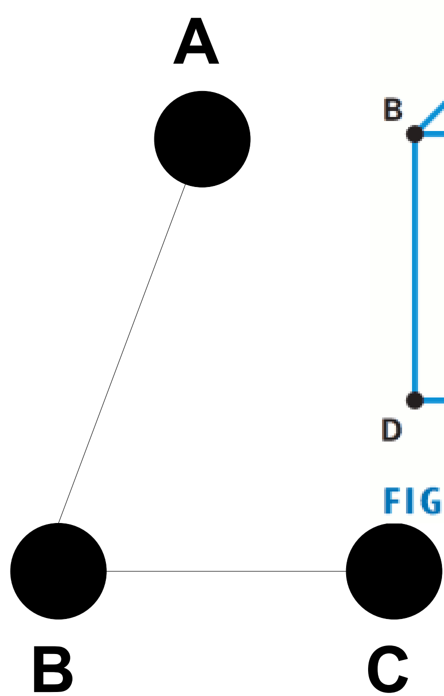
Def: A graph is **connected** if there is a path from any vertex to any other vertex.

What would it mean if the actors graph was not connected?

What would it mean if the cities graph (with roads being the edges) was not connected?

Def: a vertex is **odd** if it has an odd number of edges connected to it.

Def: a vertex is **even** it it has an even number of edges connected to it.



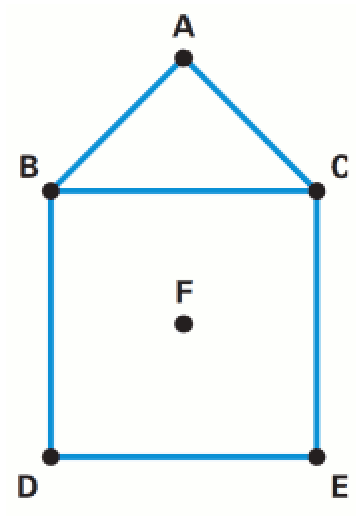
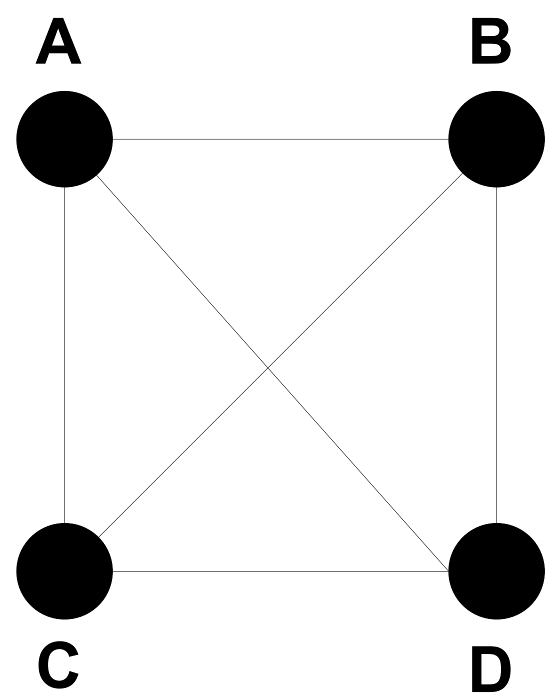


FIGURE 4.6



Def: A graph can be **traced** if there is a path that includes every edge.

This means that you can trace out the graph without picking up your pencil.

Revisiting a vertex multiple times is OK when tracing – only the edges must be crossed exactly once.

Euler's Theorem

We will now make two observations that tell us when a graph can be traced.

Observation 1: If, while tracing a graph, we neither begin nor end with vertex A, then A must be an even vertex.

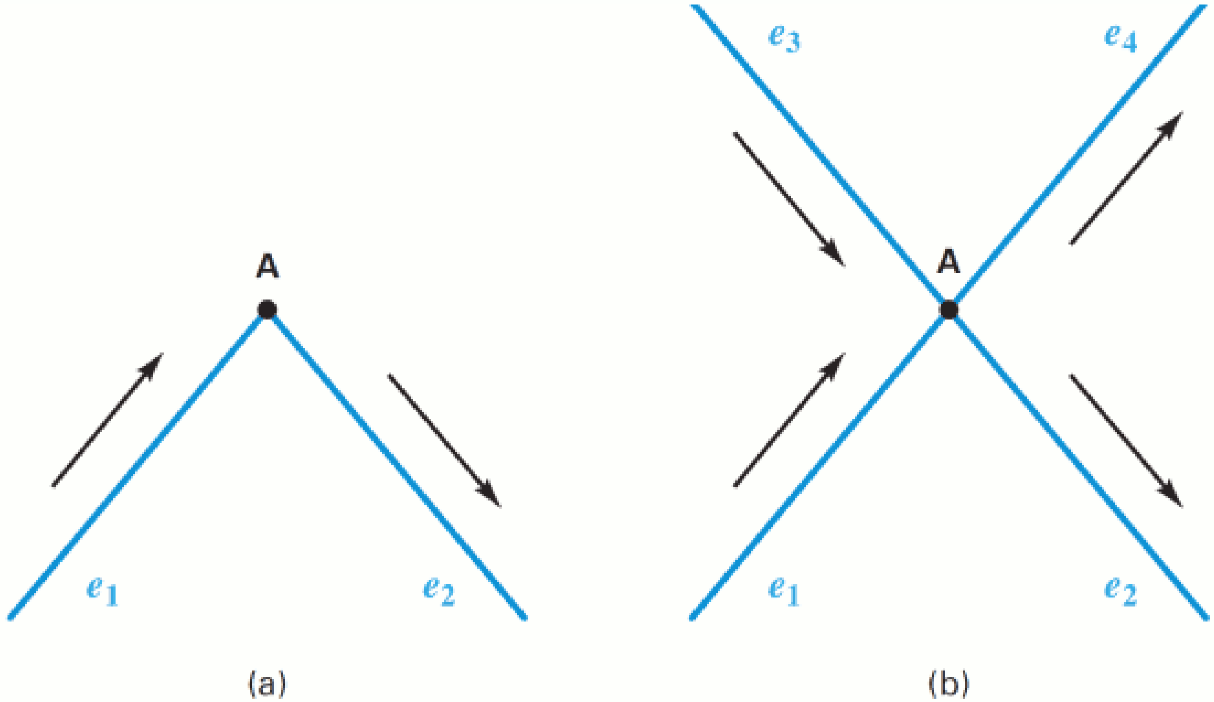


FIGURE 4.7 Every time we come into A by one edge, we must leave by another.

Euler's Theorem

Observation 2: If a graph can be traced, then it can have at most two odd vertices.

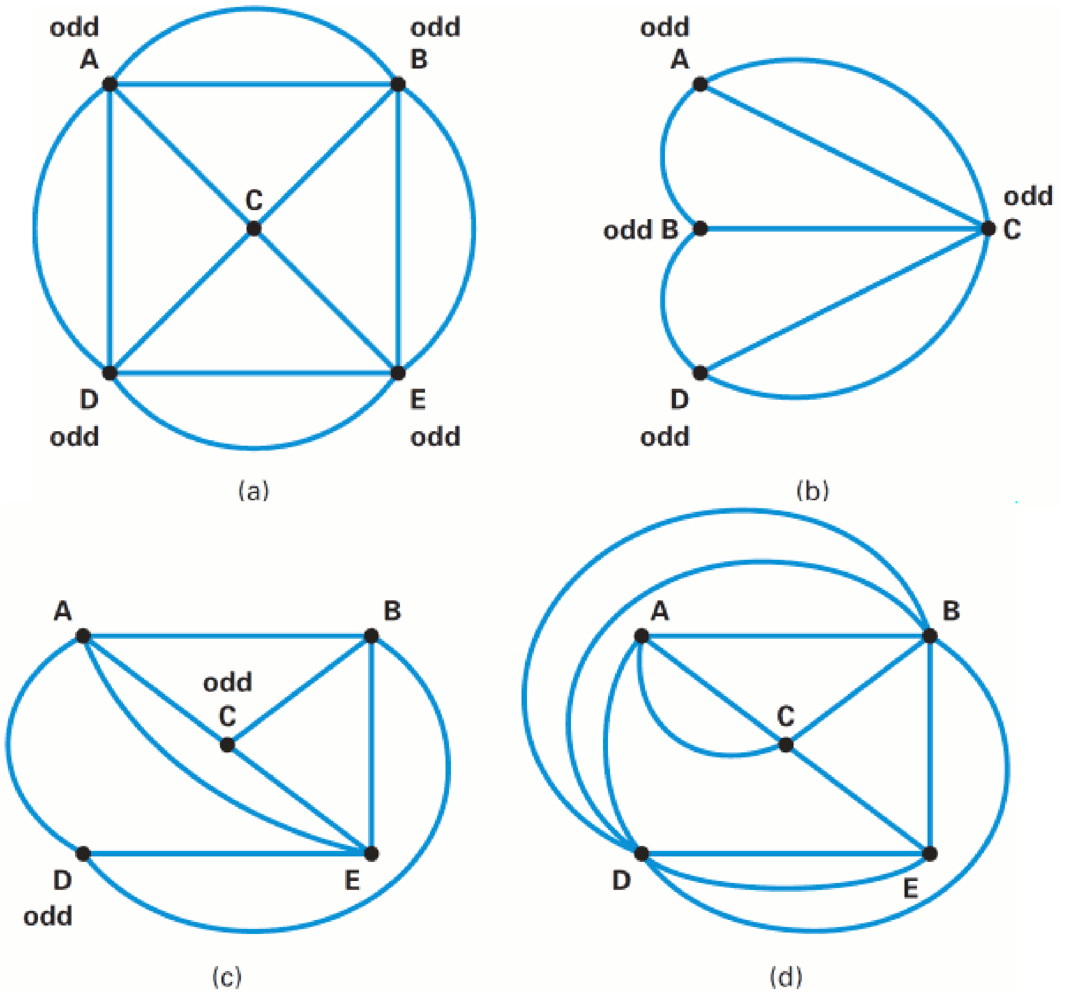
This follows directly from Observation 1. In tracing the graph, one odd vertex could be the starting vertex and another could be the ending vertex. Because no other vertices could be the starting or ending vertex, all other vertices must be even.

EULER'S THEOREM A graph can be traced if it is connected and has zero or two odd vertices.

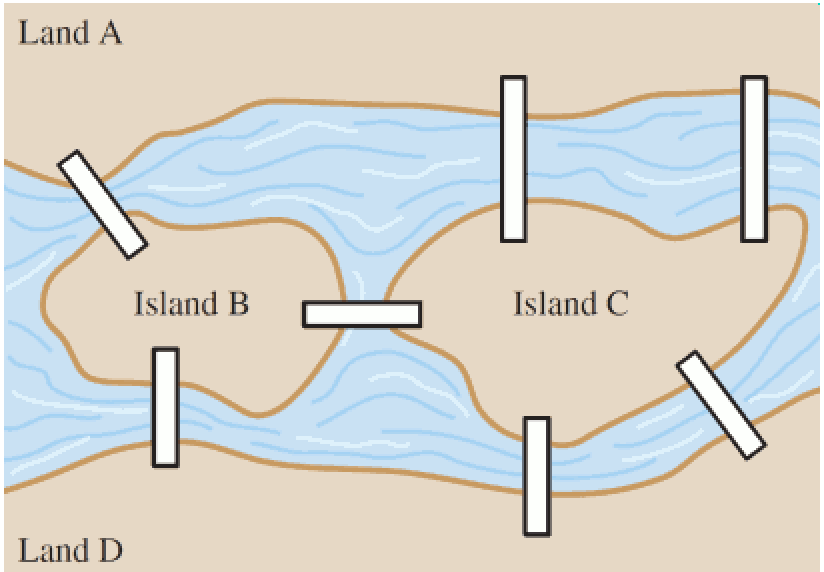
If a graph has two odd vertices, the tracing must begin at one of these and end at the other. If all the vertices are even, then the graph tracing must begin and end at the same vertex. It does not matter at which vertex this occurs.

Euler's Theorem

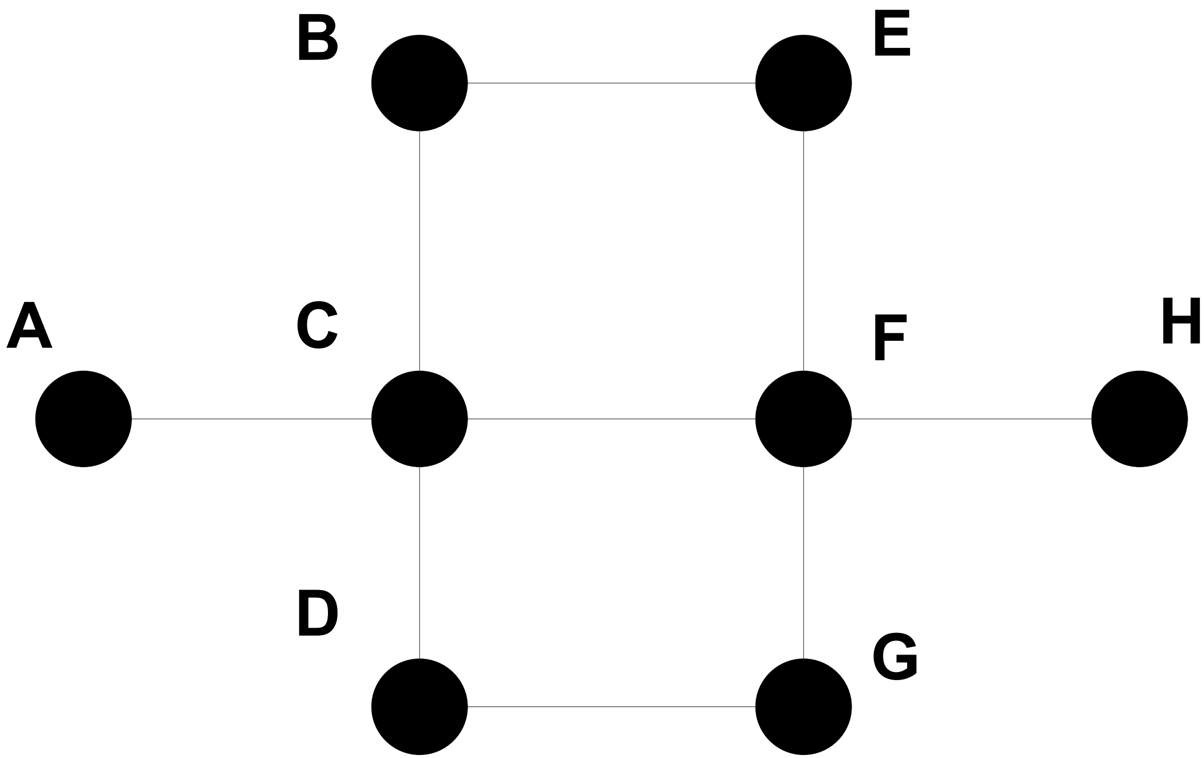
- Example: Which of the graphs can be traced?



Let's solve the Keonigsberg bridge problem!



Def: An **Euler path** is a path that traces the graph. (A path including every edge exactly once.)



Def: An **Euler circuit** is an Euler path where the start vertex and end vertex are the same.

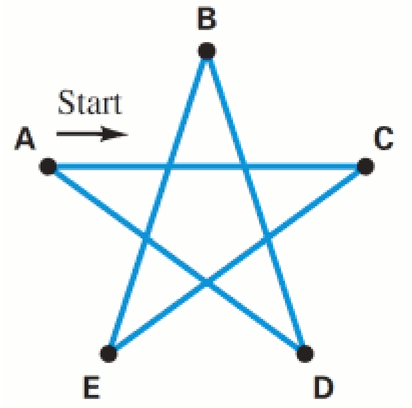
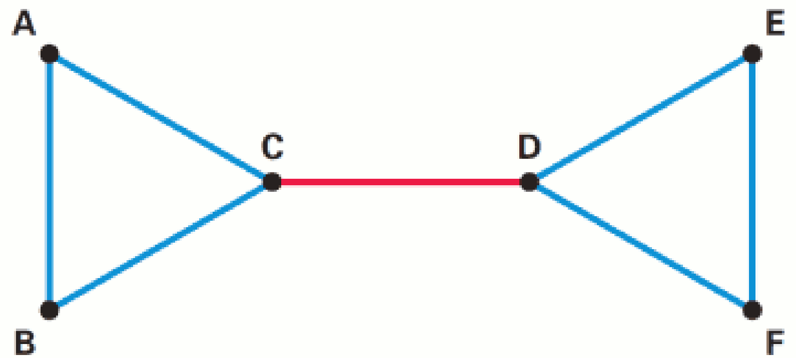


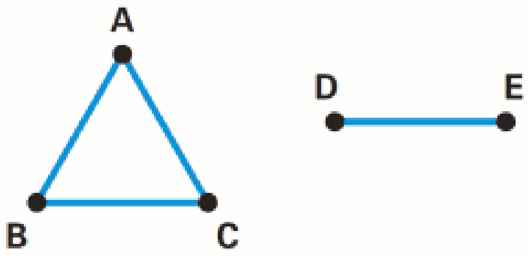
FIGURE 4.9

Graph Tracing

DEFINITIONS A graph is **connected*** if it is possible to travel from any vertex to any other vertex of the graph by moving along successive edges. A **bridge** in a connected graph is an edge such that if it were removed the graph is no longer connected.



connected graph;
edge CD is a bridge



nonconnected graph

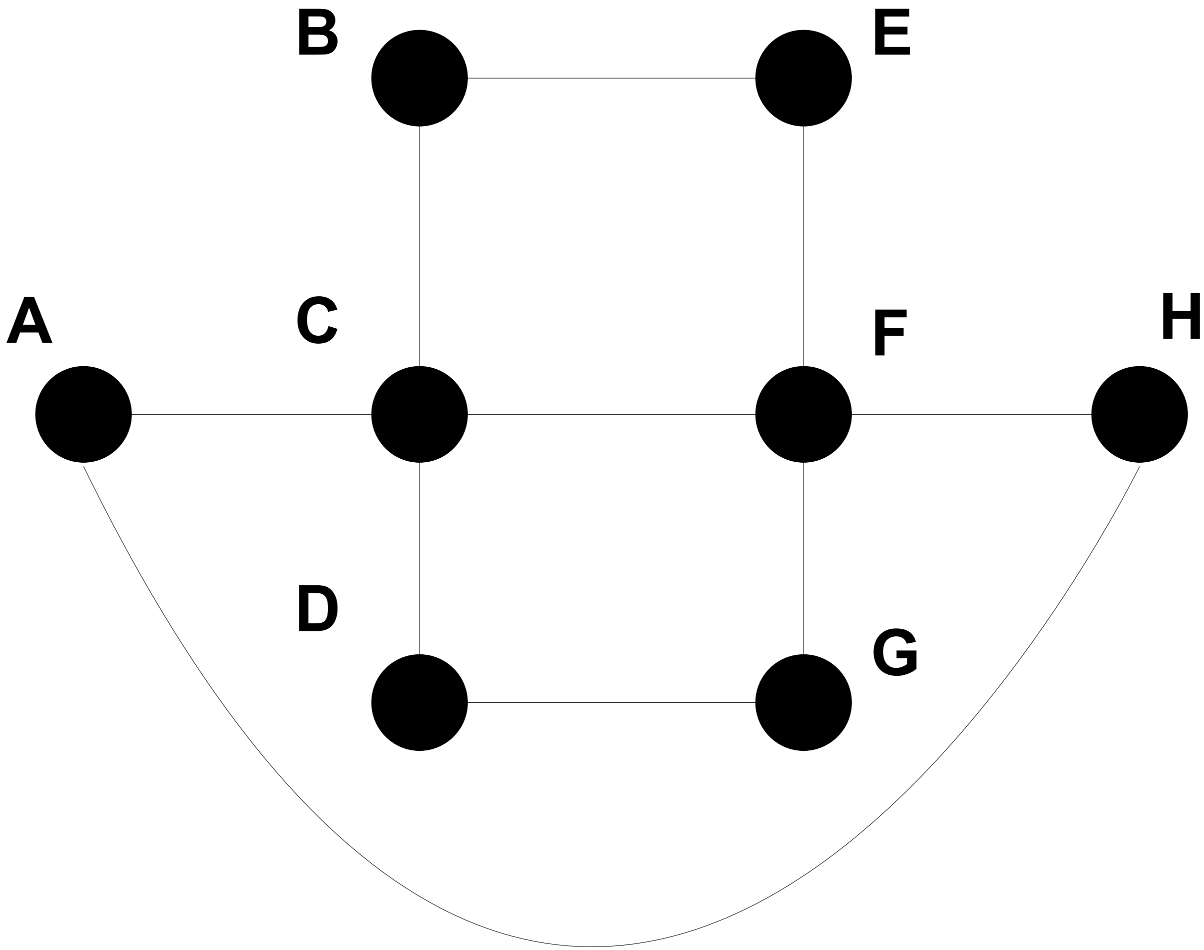
*Connected graphs are also called *networks*.

Fleury's Algorithm

- We use Fleury's algorithm to find Euler circuits.

FLEURY'S ALGORITHM If a connected graph has all even vertices, we can find an Euler circuit for it by beginning at any vertex and traveling over consecutive edges according to these rules:

1. After you have traveled over an edge, erase it. If all the edges for a particular vertex have been erased, then erase that vertex also.
2. Travel over an edge that is a bridge only if there is no alternative.



A
B
C
D
E
F
G
H

Fleury’s Algorithm

- Example:

Assume you are doing maintenance work along pathways joining locations A, B, and so on in a theme park, as shown in Figure 4.10. Find an Euler circuit in this graph to make your job efficient by not retracing pathways. Assume that you leave from and return to building C.

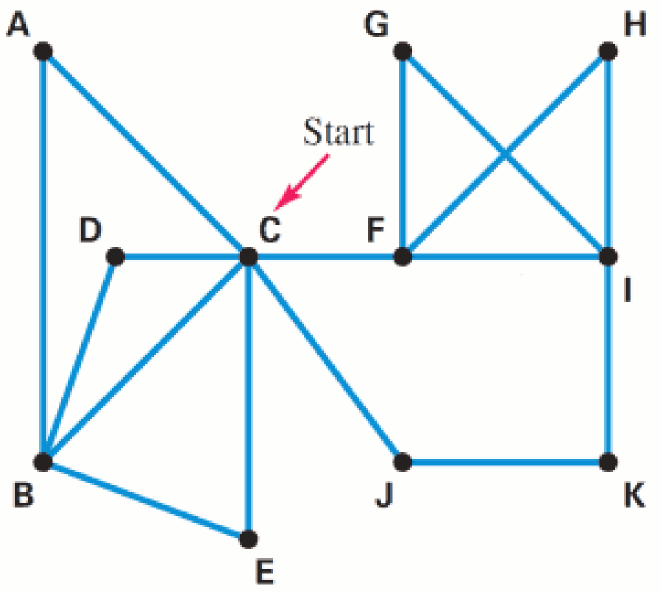


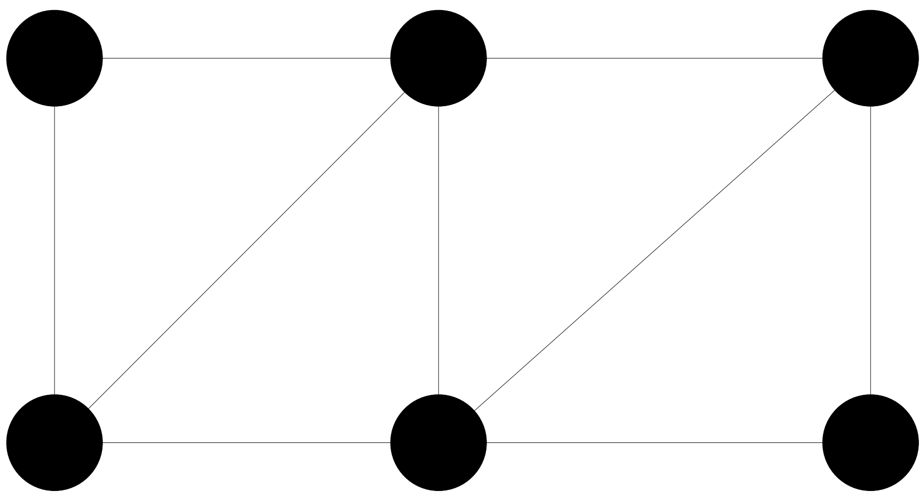
FIGURE 4.10

Eulerizing a graph is the process of adding edges so that the final graph has an Euler circuit

Which verticies cause a graph not to have an Euler path or circuit?

Answer: the odd verticies.

Eulerizing = duplicating edges until all verticies are even.



Two solutions. Which is more optimal?