13.2 Complements and Unions of Events

A sample space is a set.
An event is a set.

The idea of this section is to combine the ideas of sets with probability.

We will use complements, unions, and intersections.

Recall:
The complement of a set is the collection of elements not in that set.

A' = { elements not in A }

The complement of an event E, is the collection of outcomes not in E.

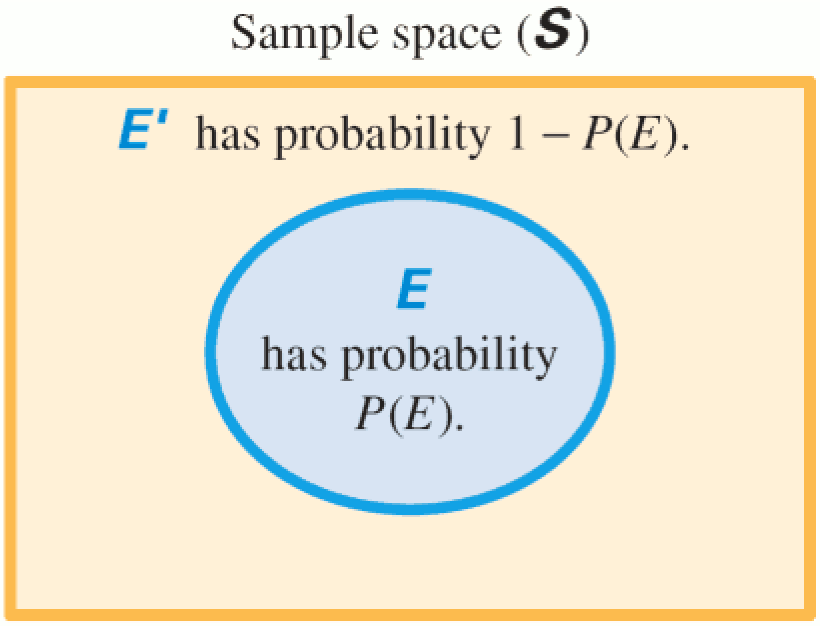
E' = { outcomes not in E }

If an outcome is in the sample space, it must be in E or E'.

So E and E' give all outcomes.

So $P(E) + P(E') = 1 \quad (100\%)$

COMPUTING THE PROBABILITY OF THE COMPLEMENT OF AN EVENT If E is an event, then $P(E') = 1 - P(E)$.



A drug was administered.

The probability that the person got better was 0.28.   (28%)

What is the probability that the person did not get better?

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The probability that the person got better was 0.28. (28%)

What is the probability that the person did not get better?

$$
\begin{aligned}
P(E') &= 1 - P(E) \\
&= 1 - 0.28 \\
&= 0.72
\end{aligned}
$$

A single card is removed from a deck.

What is the probability that it is <u>not</u> the Jack of Clubs?

A single card is removed from a deck.

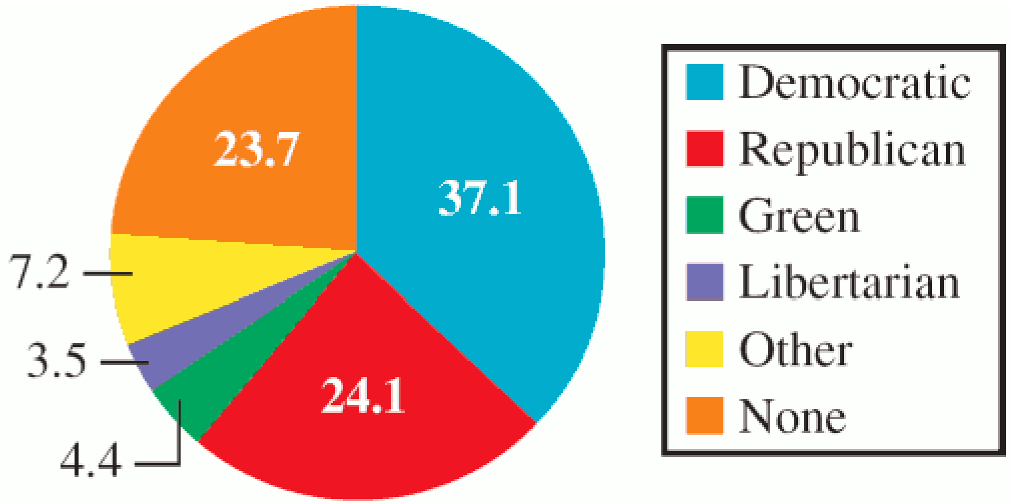
What is the probability that it is <u>not</u> the Jack of Clubs?

E = Jack of Clubs
$P(E) = 1/52$

E' = not Jack of Clubs
$P(E') = 1 – 1/52 = 52/52 – 1/52 = 51/52$

Complements of Events

- Example: The graph shows the party affiliation of a group of voters. If we randomly select a person from this group, what is the probability that the person has a party affiliation?



Percent of voters according to party affiliation

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Complements of Events

- Solution: Let *A* be the event that the person we select has some party affiliation. It is simpler to calculate the probability of *A′*. Since 23.7% have no party affiliation,

Complements of Events

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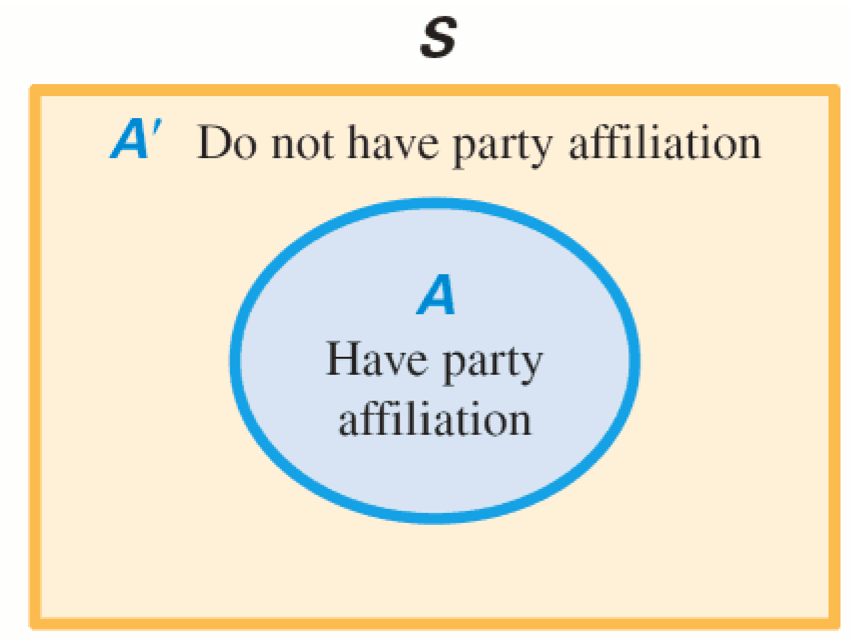


$$P(A) = 1 - P(A') = 1 - 0.237 = 0.763.$$

Word problems:

OR means union

Heart or Ace

Heart U Ace

AND means intersection

Heart and Ace

Heart ∩ Ace

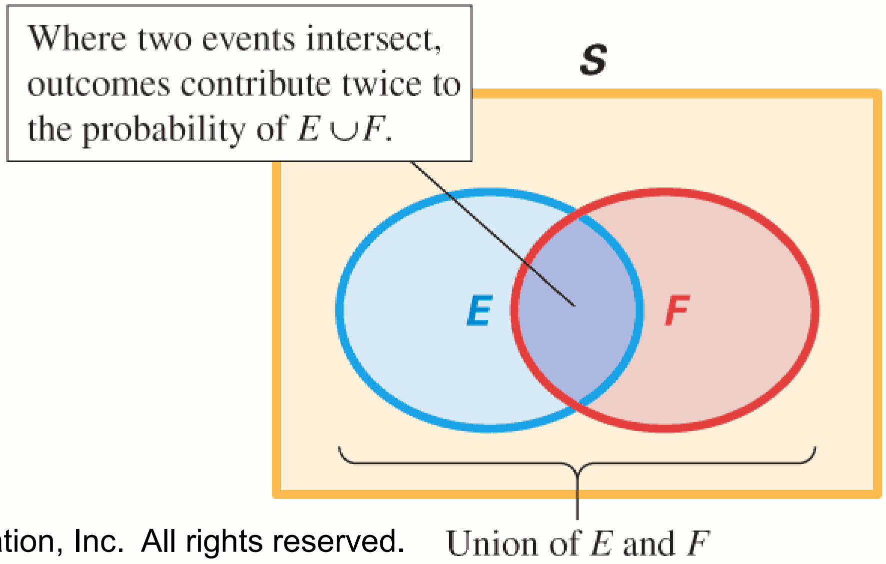
Unions of Events

RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS If E and F are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If E and F have no outcomes in common, they are called *mutually exclusive events.* In this case, because $E \cap F = \emptyset$, the preceding formula simplifies to

$$P(E \cup F) = P(E) + P(F).$$



One card was drawn from a deck.

What is the probability that it was a Heart or an Ace?

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What is the probability that it was a Heart or an Ace?

$P(\text{Heart}) = 13/52$

$P(\text{Ace}) = 4/52$

$P(\text{Heart} \cap \text{Ace}) = 1/52$ only “Ace of Hearts”

One card was drawn from a deck.

What is the probability that it was a Heart or an Ace?

$P(\text{Heart}) = 13/52$

$P(\text{Ace}) = 4/52$

$P(\text{Heart} \cap \text{Ace}) = 1/52$

$$
\begin{aligned}
P(\text{Heart} \cup \text{Ace}) &= P(\text{Heart}) + P(\text{Ace}) \\
&\quad - P(\text{Heart} \cap \text{Ace}) \\
&= 16/52 = 4/13
\end{aligned}
$$

Unions of Events

- Example: If we select a single card from a standard 52-card deck, what is the probability that we draw either a heart or a face card?

- Solution: Let *H* be the event "draw a heart" and *F* be the event "draw a face card." We are looking for $P(H \cup F)$.

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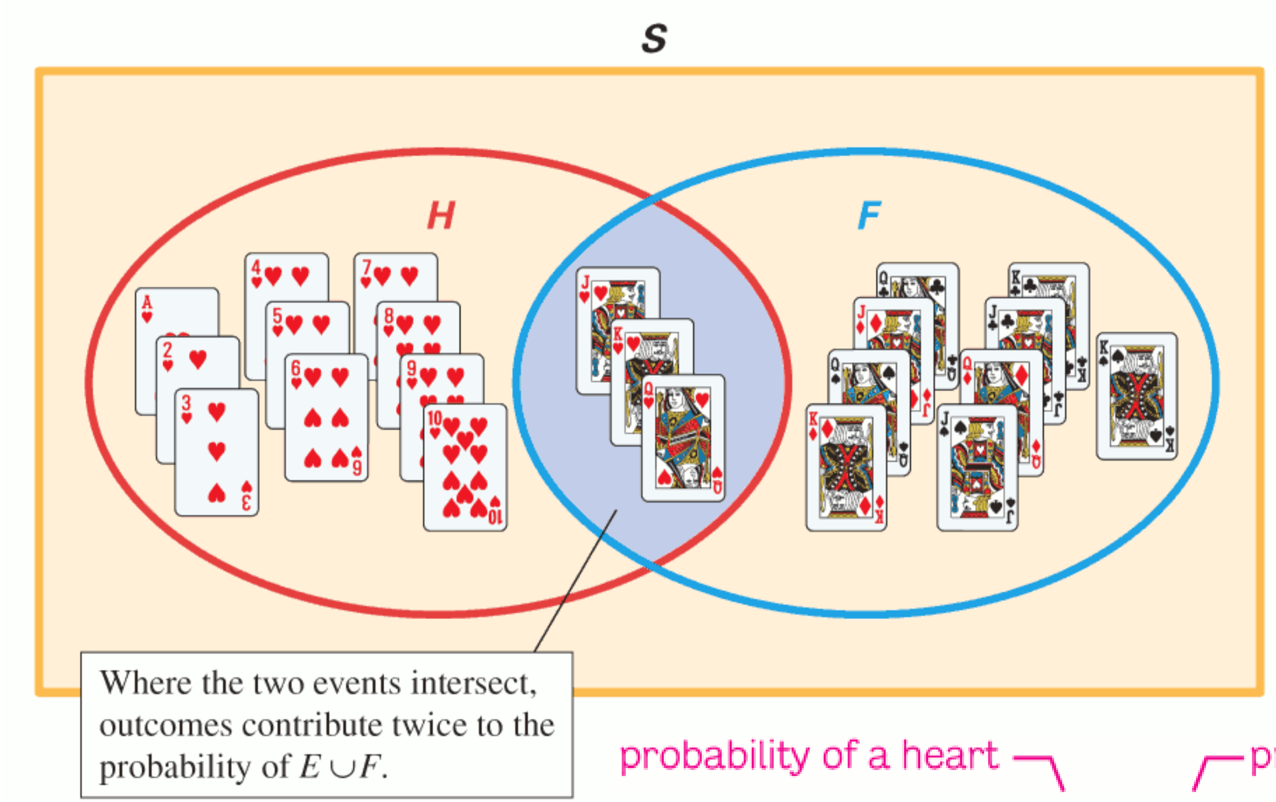
Unions of Events



There are 13 hearts, 12 face cards, and 3 cards that are both hearts and face cards.

$$P(H \cup F) = P(H) + P(F) - P(H \cap F) = \overbrace{\frac{13}{52}}^{\text{probability of a heart}} + \overbrace{\frac{12}{52}}^{\text{probability of a face card}} - \underbrace{\frac{3}{52}}_{\text{probability of a heart that is a face card}} = \frac{22}{52} = \frac{11}{26}$$

If you are given 3 out of the 4 terms in the equation

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Then you can use algebra to find the remaining term.

This can also be read as

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

The probability a UT student is

- an Education major is 0.09.
- an Ed. major and in athletics is 0.01
- in Ed or athletics is 0.12.

What is the probability that a UT student is in athletics?

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What is the probability that a UT student is in athletics?

$$P(\text{Ed or ath}) = P(\text{Ed}) + P(\text{ath}) – P(\text{Ed or ath})$$
$$0.12 = 0.09 + P(\text{ath}) – 0.01$$
$$0.12 = 0.08 + P(\text{ath})$$
$$0.04 = P(\text{ath})$$

The probability of a person being
- happy is 0.45
- a millionaire is 0.02
- happy or a millionaire is 0.46

What is the probability that a person is happy and a millionaire?

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- happy is 0.45
- a millionaire is 0.02
- happy or a millionaire is 0.46

What is the probability that a person is happy and a millionaire?

P(happy or \$\$\$) =

P(happy) + P(\$\$\$) - P(happy and \$\$\$)

0.46 = 0.45 + 0.02 – P(H and \$\$\$)

0.46 = 0.47 – P(H and \$\$\$)

P(H and \$\$\$) = 0.01