1. Find $\frac{dy}{dx}$ by applying the chain rule repeatedly. Show your work. (6 points)

$$y = (\sqrt{1-3x^2}+1)^3$$

$g(x) = x^3 \qquad h(x) = \sqrt{1-3x^2}+1$

$h(x) = \sqrt{1-3x^2}+1$

$h'(x) = \frac{1}{2}(1-3x^2)^{-1/2} \cdot (-6x)$

$= \frac{-3x}{(1-3x^2)^{1/2}}$

$\frac{dy}{dx} = 3(\sqrt{1-3x^2}+1)^2 \cdot h'(x)$

$= 3(\sqrt{1-3x^2}+1)^2 \cdot \frac{(-3x)}{\sqrt{1-3x^2}}$

$= \boxed{\frac{-9x(\sqrt{1-3x^2}+1)^2}{\sqrt{1-3x^2}}}$

2. Find the derivative of (6 points)

$$f(x) = \sin\sqrt{2x^2-x+1}$$

Show your work.

$g(x) = \sin x \qquad h(x) = \sqrt{2x^2-x+1}$

$f'(x) = \cos\sqrt{2x^2-x+1} \cdot h'(x)$

$= \cos\sqrt{2x^2-x+1} \cdot \frac{1}{2}(2x^2-x+1)^{-1/2} \cdot (4x-1)$

$= \boxed{\frac{(4x-1) \cdot \cos\sqrt{2x^2-x+1}}{2\sqrt{2x^2-x+1}}}$

3. Find $\frac{dy}{dx}$ by **implicit differentiation**. Then find the equation of the **normal** line to the curve $y = f(x)$ at the point (1, 2). Show your work. (10 points)

$$2x^2 - xy - 2y^3 = -16$$

$$2x^2 - xy - 2y^3 = -16$$

$$\frac{d}{dx}(2x^2) - \frac{d}{dx}(xy) - \frac{d}{dx}(2y^3) = \frac{d}{dx}(-16)$$

$$4x - \left[x\frac{dy}{dx} + y \cdot \frac{dx}{dx}\right] - 6y^2 \cdot \frac{dy}{dx} = 0$$

$$4x - x\frac{dy}{dx} - y - 6y^2\frac{dy}{dx} = 0$$

$$4x - y = x\frac{dy}{dx} + 6y^2\frac{dy}{dx}$$

$$4x - y = \frac{dy}{dx}(x + 6y^2)$$

$$\boxed{\frac{dy}{dx} = \frac{4x-y}{x+6y^2}}$$

Slope of the tangent line at $(1,2) = \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{4.1-2}{1+6.2^2}$

$$= \frac{4-2}{1+24} = \frac{2}{25}$$

Slope of the normal line $= -\frac{25}{2}$

<u>Equation of the normal line</u>

$$y - 2 = -\frac{25}{2}(x-1)$$

$$y = -\frac{25x}{2} + \frac{25}{2} + 2$$

$$\boxed{y = \frac{-25}{2}x + \frac{29}{2}}$$

4. Differentiate the following functions. Show your work. (8 points)

(a) $f(x) = e^{\sin(x^2-1)}$

(b) $f(x) = \ln(\cos x)$

(a) $g(x) = e^x$, $h(x) = \sin(x^2-1)$

$f'(x) = e^{\sin(x^2-1)} \cdot h'(x)$

$= \boxed{e^{\sin(x^2-1)} \cdot \cos(x^2-1) \cdot 2x}$

(b) $g(x) = \ln x$, $h(x) = \cos x$

$f'(x) = \frac{1}{\cos x} \cdot h'(x)$

$= \frac{1}{\cos x} \cdot (-\sin x)$

$= -\frac{\sin x}{\cos x}$

$= \boxed{-\tan x}$

5. Let

$$f(x) = x - \sin x.$$

Find $\frac{d}{dx} f^{-1}(x)\Big|_{x=\pi}$. [Note that $f(\pi) = \pi$]. Show your work. (6 points)

$\frac{d}{dx} f^{-1}(x)\Big|_{x=\pi} = \frac{1}{f'(f^{-1}(\pi))}$

$= \frac{1}{f'(\pi)}$

$= \boxed{\frac{1}{2}}$

$f'(x) = 1 - \cos x$

$f'(\pi) = 1 - \cos \pi$

$= 1 - (-1) = 2$

6. Use **logarithmic differentiation** to find the derivative of the function. Show your work. (6 points)

$$f(x) = (\ln x)^{3x}$$

$$y = (\ln x)^{3x}$$

$$\ln y = 3x \cdot \ln(\ln x)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(3x \cdot \ln(\ln x))$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 3x \cdot \frac{d}{dx}(\ln(\ln x)) + \ln(\ln x) \cdot \frac{d}{dx}(3x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 3x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + 3\ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{\ln x} + 3\ln(\ln x)$$

$$\frac{dy}{dx} = y\left[\frac{3}{\ln x} + 3\ln(\ln x)\right]$$

$$\boxed{\frac{dy}{dx} = (\ln x)^{3x}\left[\frac{3}{\ln x} + 3\ln(\ln x)\right]}$$

7. Use the formula of linear approximation at $x = a$

$$L(x) = f(a) + f'(a)(x - a)$$

to approximate the value of the given function. Then compare your result with the value you get from a calculator. Show your work. (Hint : Think about what function to begin with depending on the expression below). (8 points)

$$(1.01)^{25}$$

$f(x) = x^{25}$, $a = 1$

$f'(x) = 25x^{24}$

$f'(1) = 25$

$L(x) = f(1) + f'(1)(x-1)$

$= 1 + 25(x-1)$

$= 25x - 24$

$L(1.01) = 25(1.01) - 24$

$= 25.25 - 24$

$= \boxed{1.25}$

Using calculator

$(1.01)^{25} = 1.28$