Chapter 7

Math 2890-001
Spring 2018 **Name** ____________________
Due May 02

1. (3 points) Explain your answer for each part of this question.

   (a) Write down a 4×4 matrix (with no zero entries) that you know has real eigenvalues, and that can be diagonalized with an orthogonal similarity transformation.

   (b) Now change some of the entries in your matrix to zeros to get a matrix with real eigenvalues that cannot be diagonalized with an orthogonal similarity transformation.

   (c) Determine whether your matrix from part (b) can be diagonalized by some similarity transformation.

2. (1 point) Let

$$A = \begin{pmatrix} -5 & -1 & 5 \\ 6 & 3 & -6 \\ -5 & 8 & 3 \end{pmatrix}.$$

Find an orthogonal matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

3. (1 point) Let

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 4 & -2 \\ 4 & -2 & 1 \end{pmatrix}.$$

Find an orthogonal matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

HINT: The eigenvalues of A are $5, 5, -4$.

4. (1 point) Let

$$A = \begin{pmatrix} 3 & 0 & 1 & 2 \\ 0 & 3 & -2 & -1 \\ 1 & -2 & 3 & 0 \\ 2 & -1 & 0 & 3 \end{pmatrix}.$$

Find an orthogonal matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

HINT: The eigenvalues of A are $6, 4, 2, 0$.

5. (1 point) Let

$$Q(x) = 3x_1^2 + 4x_1x_2 - 6x_1x_3 + 9x_2^2 + 12x_2x_3 + 5x_3^2.$$

Find the (symmetric) matrix of the quadratic form $Q(x)$.

6. (1 point) Let

$$A = \begin{pmatrix} -8 & 8 & -2 \\ 8 & -8 & -3 \\ -2 & -3 & 4 \end{pmatrix}.$$

Find the quadratic form $Q(x) = x^T Ax$.

7. (1 point) Let

$$A = \begin{pmatrix} 4 & 4 \\ 4 & 8 \end{pmatrix}.$$

Find the maximum value of the quadratic form $Q(x) = x^T Ax$ subject to the constraint $x^T x = 1$.

HINT: The characteristic polynomial of A is $\lambda^2 - 12\lambda + 16$.

8. (1 point) Let

$$A = \begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}.$$

Find the vector x that maximizes the quadratic form $Q(x) = x^T Ax$ subject to the constraint $x^T x = 1$.

HINT: The characteristic polynomial of A is $\lambda^2 - 7\lambda + 6$.

9. (1 point) Let

$$U = \begin{pmatrix} 2/9 & -4/9 & -5/9 \\ -5/9 & -2/3 & -2/9 \\ 4/9 & 2/9 & -2/3 \\ 2/3 & -5/9 & 4/9 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$V = \begin{pmatrix} -2/7 & 3/7 & 6/7 \\ 3/7 & 6/7 & -2/7 \\ -6/7 & 2/7 & -3/7 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 8 \\ 4 \\ 0 \\ 2 \end{pmatrix}.$$

Use the reduced singular value decomposition $A = U\Sigma V^T$ to find a least-squares solution of $Ax = b$ having minimal 2-norm.

10. (1 point) Let $A = \begin{pmatrix} 10 & -8 & -6 \\ 8 & -12 & -2 \\ -6 & 2 & 5 \end{pmatrix}$.

Find the reduced singular value decomposition of A.

HINT:

$$A^T A = \begin{pmatrix} 200 & -188 & -106 \\ -188 & 212 & 82 \\ -106 & 82 & 65 \end{pmatrix}$$

and this has nonzero eigenvalues $441 = 21^2, 36 = 6^2$.

Total for assignment: 12 points