Chapter 6

Math 2890-001
Spring 2018 **Name** _____
Due Mar 28

1. (1 point) Let

$$u = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 0 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 7 \\ 3 \\ 4 \\ 1 \end{pmatrix}.$$

Find the inner product $u \cdot v$. Show your work.

2. (1 point) Let

$$v = \begin{pmatrix} 6 \\ -4 \\ -2 \\ 5 \end{pmatrix}.$$

Find a unit vector in the direction of v. Show your work.

3. (1 point) Let

$$u = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 3 \\ -6 \\ -4 \\ 2 \end{pmatrix}.$$

Find the distance between u and v. Show and explain your computations.

4. (1 point) Let

$$u_1 = \begin{pmatrix} 1 \\ -5 \\ 5 \\ -1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 5 \\ -3 \\ -3 \\ 5 \end{pmatrix} \quad \text{and} \quad u_3 = \begin{pmatrix} 7 \\ 7 \\ 5 \\ -3 \end{pmatrix}.$$

Is the set $\{u_1, u_2, u_3\}$ orthogonal? Why or why not? Show your computations.

5. (1 point) Let

$$y = \begin{pmatrix} 2 \\ 0 \\ 7 \\ 1 \end{pmatrix}$$

and let W be the span of

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}.$$

Project y onto W. Show and explain your computations.

6. (1 point) Let

$$y = \begin{pmatrix} 2 \\ 0 \\ 7 \\ 1 \end{pmatrix}$$

and let W be the span of

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}.$$

Find the point in W that is closest to y. Show and explain your computations.

7. (1 point) Let

$$y = \begin{pmatrix} 2 \\ 0 \\ 7 \\ 1 \end{pmatrix}$$

and let W be the span of

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}.$$

Write y as a sum of a vector in W and a vector orthogonal to W. Show and explain your computations.

8. (1 point) Let

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 2 \\ 1 & 2 \\ -1 & -1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 7 \\ 7 \end{pmatrix}.$$

Find the least squares solution to $Ax = b$. Show and explain your computations.

9. (1 point) Let

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 2 \\ 1 & 2 \\ -1 & -1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 7 \\ -2 \\ 1 \\ 2 \\ 2 \end{pmatrix}.$$

Find the least squares error in the least squares solution to $Ax = b$. Show and explain your computations.

HINT: The least squares solution is $x = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

10. (1 point) Let

$$Q = \begin{pmatrix} 2/9 & 4/9 \\ -5/9 & 6/9 \\ -4/9 & 2/9 \\ 6/9 & 5/9 \end{pmatrix}, \; R = \begin{pmatrix} 5 & -11 \\ 0 & 3 \end{pmatrix} \text{ and } b = \begin{pmatrix} 11 \\ -1 \\ 3 \\ 2 \end{pmatrix}.$$

Use the QR factorization $A = QR$ to find the least squares solution to $Ax = b$.

Show your work.

11. (1 point) Let

$$A = \begin{pmatrix} 0 & -1 & 6 \\ -1 & 4 & -9 \\ -3 & 9 & -12 \\ 1 & -3 & 3 \\ 1 & -2 & 0 \end{pmatrix}.$$

Find the QR factorization of A.

Show and explain your computations.

12. (1 point) Use the QDR factorization

$$A = \begin{pmatrix} -2 & 0 & 8 \\ -1 & -2 & -6 \\ -2 & -4 & -7 \\ 1 & 0 & -4 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} -2 & 2 & 0 \\ -1 & -1 & -2 \\ -2 & -2 & 1 \\ 1 & -1 & 0 \end{pmatrix}}_{Q} \underbrace{\begin{pmatrix} 1/10 & 0 & 0 \\ 0 & 1/10 & 0 \\ 0 & 0 & 1/5 \end{pmatrix}}_{D} \underbrace{\begin{pmatrix} 10 & 10 & 0 \\ 0 & 10 & 40 \\ 0 & 0 & 5 \end{pmatrix}}_{R}$$

to find the least squares solution to $Ax = b$ where $b = \begin{pmatrix} 10 \\ -5 \\ 10 \\ -15 \end{pmatrix}$.

13. (1 point) Let

$$A = \begin{pmatrix} -2 & 4 & -12 \\ 3 & -2 & -1 \\ 0 & 2 & -7 \\ 1 & 0 & -1 \\ -4 & 4 & -10 \end{pmatrix}.$$

Find the QDR factorization of A.

Show and explain your computations.

14. (1 point) Consider the data points $(1, -3), (2, -6), (3, 9), (4, 1)$.

    Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the given data points.

    Show and explain your computations.

15. (1 point) Consider the data points $(1, -5), (2, -7), (3, -4), (4, 5)$.

    Find the equation $y = \beta_0 + \beta_1 x + \beta_2 x^2$ of the least-squares quadratic that best fits the given data points.

    Show and explain your computations.

Total for assignment: 15 points