

Chapter 7

Math 2890-003

Spring 2016

Due May 05

Name _____

1. (3 points) Explain your answer for each part of this question.
 - (a) Write down a 4×4 matrix (with no zero entries) that you know has real eigenvalues, and that can be diagonalized with an orthogonal similarity transformation.
 - (b) Now change some of the entries in your matrix to zeros to get a matrix with real eigenvalues that cannot be diagonalized with an orthogonal similarity transformation.
 - (c) Determine whether your matrix from part (b) can be diagonalized by some similarity transformation.

2. (1 point) Let

$$A = \begin{pmatrix} -5 & -1 & 5 \\ 6 & 3 & -6 \\ -5 & 8 & 3 \end{pmatrix}.$$

Find an orthogonal matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

3. (1 point) Let

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 4 & -2 \\ 4 & -2 & 1 \end{pmatrix}.$$

Find an orthogonal matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

HINT: The eigenvalues of A are $5, 5, -4$.

4. (1 point) Let

$$A = \begin{pmatrix} 3 & 0 & 1 & 2 \\ 0 & 3 & -2 & -1 \\ 1 & -2 & 3 & 0 \\ 2 & -1 & 0 & 3 \end{pmatrix}.$$

Find an orthogonal matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

HINT: The eigenvalues of A are 6, 4, 2, 0.

5. (1 point) Let

$$Q(x) = 3x_1^2 + 4x_1x_2 - 6x_1x_3 + 9x_2^2 + 12x_2x_3 + 5x_3^2.$$

Find the (symmetric) matrix of the quadratic form $Q(x)$.

6. (1 point) Let

$$A = \begin{pmatrix} -8 & 8 & -2 \\ 8 & -8 & -3 \\ -2 & -3 & 4 \end{pmatrix}.$$

Find the quadratic form $Q(x) = x^T Ax$.

7. (1 point) Let

$$A = \begin{pmatrix} 4 & 4 \\ 4 & 8 \end{pmatrix}.$$

Find the maximum value of the quadratic form $Q(x) = x^T A x$ subject to the constraint $x^T x = 1$.

8. (1 point) Let

$$A = \begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}.$$

Find the vector x that maximizes the quadratic form $Q(x) = x^T A x$ subject to the constraint $x^T x = 1$.

9. (1 point) Let

$$U = \begin{pmatrix} 2/9 & -4/9 & -5/9 \\ -5/9 & -2/3 & -2/9 \\ 4/9 & 2/9 & -2/3 \\ 2/3 & -5/9 & 4/9 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$V = \begin{pmatrix} -2/7 & 3/7 & 6/7 \\ 3/7 & 6/7 & -2/7 \\ -6/7 & 2/7 & -3/7 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 8 \\ 4 \\ 0 \\ 2 \end{pmatrix}.$$

Use the reduced singular value decomposition $A = U\Sigma V^T$ to find a least-squares solution of $Ax = b$ having minimal 2-norm.

10. (1 point) Let

$$A = \begin{pmatrix} 11 & 2 & 10 \\ 6 & 24 & -6 \\ 26 & 20 & 16 \end{pmatrix}.$$

Find the reduced singular value decomposition of A .

HINT:

$$A^T A = \begin{pmatrix} 833 & 686 & 490 \\ 686 & 980 & 196 \\ 490 & 196 & 392 \end{pmatrix}$$

and this has nonzero eigenvalues 1764, 441.

Total for assignment: 12 points