# Chapter 4

**Math 2890-003**
**Spring 2016** **Name** ______________________________
**Due Mar 31**

1. (1 point) Let

$$\{y_0, y_1, y_2, \ldots\} = \{-3, 4, 2, -5, 5, 2, 0, -5, -4, -3, 3, 0, 5, \ldots\}.$$

Use the filter

$$z_k = 4y_{k+3} - 3y_{k+2} - 4y_{k+1} + 3y_k$$

to find the first 6 terms of the signal $\{z_0, z_1, z_2, \ldots\}$. Show your work.

answer: $\{z_0, z_1, z_2, \ldots\} = \{-51, 39, 19, -41, -13, 5, \ldots\}$

2. (1 point) Let

$$A = \begin{pmatrix} -154 & 112 & 58 & -36 \\ -183 & 133 & 68 & -42 \\ -198 & 144 & 77 & -48 \\ -226 & 164 & 86 & -53 \end{pmatrix}.$$

Compute $A^9$. Show and explain your work.

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & 4 & 2 & 2 \\ 1 & 5 & 4 & 1 \\ 2 & 4 & -3 & 9 \\ 2 & 5 & -1 & 9 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad P^{-1} = \begin{pmatrix} -43 & 32 & 18 & -12 \\ 14 & -10 & -5 & 3 \\ -7 & 5 & 2 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

answer:

$$\begin{aligned} A^9 &= PD^9P^{-1} \\ &= \begin{pmatrix} 1 & 4 & 2 & 2 \\ 1 & 5 & 4 & 1 \\ 2 & 4 & -3 & 9 \\ 2 & 5 & -1 & 9 \end{pmatrix} \begin{pmatrix} 512 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -43 & 32 & 18 & -12 \\ 14 & -10 & -5 & 3 \\ -7 & 5 & 2 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -22084 & 16432 & 9238 & -6156 \\ -22113 & 16453 & 9248 & -6162 \\ -44058 & 32784 & 18437 & -12288 \\ -44086 & 32804 & 18446 & -12293 \end{pmatrix}. \end{aligned}$$

3. (1 point) Let

$$A = \begin{pmatrix} 0.6 & 0.2 & 0.5 & 0.1 \\ 0.1 & 0.6 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.3 & 0.1 \\ 0.2 & 0.1 & 0.1 & 0.5 \end{pmatrix}.$$

Find a steady state probability vector for the stochastic matrix $A$. Show and expain your work.

answer: A steady state probability vector $x$ has to be a solution of

$$(A - I)x = 0$$

and have nonnegative entries that sum to 1. Since

$$A - I = \begin{pmatrix} -0.4 & 0.2 & 0.5 & 0.1 \\ 0.1 & -0.4 & 0.1 & 0.3 \\ 0.1 & 0.1 & -0.7 & 0.1 \\ 0.2 & 0.1 & 0.1 & -0.5 \end{pmatrix} \text{ we get } x = \begin{pmatrix} 0.3581 \\ 0.2905 \\ 0.125 \\ 0.2264 \end{pmatrix}$$

after row reducing and making an appropriate choice of free variable.

4. (1 point) Consider

$$y_{k+2} - 4y_{k+1} - 12y_k = 0.$$

Find the general solution of this linear difference equation. Show and explain your work.

answer: The general solution is $y_k = c_1(6)^k + c_2(-2)^k$.

5. (1 point) Consider

$$y_{k+2} + 11y_{k+1} + 28y_k = 200.$$

Find the general solution of this linear difference equation. Show and explain your work.

answer: The general solution is $y_k = c_1(-4)^k + c_2(-7)^k + 5$.

6. (1 point) Consider
$$y_{k+2} - 25y_k = -36(-4)^k.$$
Find the general solution of this linear difference equation. Show and explain your work.

answer: The general solution is $y_k = c_1(-5)^k + c_2(5)^k + 4(-4)^k$.

7. (1 point) Consider

$$y_{k+2} + 5y_{k+1} - 6y_k = 28k + 4.$$

Find the general solution of this linear difference equation. Show and explain your work.

answer: The general solution is $y_k = c_1(-6)^k + c_2(1)^k + 2k^2 - 2k$.

Total for assignment: 7 points