

Chapter 3

Math 2890-003

Spring 2016

Due Mar 24

Name _____

1. (1 point) Let

$$A = \begin{pmatrix} -8 & 0 & 0 & 0 & 0 & 0 \\ -9 & 6 & 0 & 0 & 0 & 0 \\ 7 & -2 & 1 & 0 & 0 & 0 \\ -5 & -2 & 5 & -4 & 0 & 0 \\ 0 & 2 & 2 & -1 & 1 & 0 \\ 9 & 9 & 2 & 8 & -1 & 2 \end{pmatrix}.$$

Find the determinant of A if it exists. Show and explain your computations. If the determinant doesn't exist, explain why it doesn't.

answer: The determinant of a triangular matrix is the product of the diagonal entries of the matrix, so $\det(A) = (-8)(6)(1)(-4)(1)(2) = 384$.

2. (1 point) Let

$$A = \begin{pmatrix} 3 & 5 & 6 & 4 & 0 & 1 & 0 & 4 \\ 1 & 1 & 7 & 2 & 0 & 6 & 5 & 0 \\ 0 & 0 & 9 & 2 & 4 & 1 & 2 & 3 \\ 0 & 0 & 3 & 7 & 4 & 6 & 7 & 2 \\ 0 & 0 & 0 & 0 & 0 & 8 & 9 & 4 \\ 0 & 0 & 0 & 0 & 5 & 9 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 8 \end{pmatrix}.$$

Find the determinant of A if it exists. Show and explain your computations. If the determinant doesn't exist, explain why it doesn't.

answer: The determinant of a block triangular matrix is the product of the determinants of the diagonal blocks, so

$$\begin{aligned} \det(A) &= \det \begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix} \cdot \det \begin{pmatrix} 9 & 2 \\ 3 & 7 \end{pmatrix} \cdot \det \begin{pmatrix} 0 & 8 \\ 5 & 9 \end{pmatrix} \cdot \det \begin{pmatrix} 9 & 7 \\ 3 & 8 \end{pmatrix} \\ &= ((3)(1) - (5)(1)) \cdot ((9)(7) - (2)(3)) \cdot ((0)(9) - (8)(5)) \cdot ((9)(8) - (7)(3)) \\ &= (-2)(57)(-40)(51) \\ &= 232560 \end{aligned}$$

3. (1 point) Let

$$A = \begin{pmatrix} 2 & 8 & 1 & 0 \\ 1 & 8 & 3 & 3 \\ 3 & 6 & 1 & 1 \end{pmatrix}.$$

Find the determinant of A if it exists. Show and explain your computations. If the determinant doesn't exist, explain why it doesn't.

answer: Only square matrices have determinants. The matrix A isn't square so it doesn't have a determinant.

4. (1 point) Let

$$A = \begin{pmatrix} -4 & 0 & 0 & 0 & 0 \\ 4 & -1 & 0 & 0 & 0 \\ -4 & 4 & 2 & 0 & 0 \\ -3 & 5 & -3 & -1 & 0 \\ -3 & -1 & 0 & -5 & -3 \end{pmatrix} \begin{pmatrix} 4 & -5 & -2 & 1 & 2 \\ 0 & -2 & -3 & -5 & 2 \\ 0 & 0 & -1 & 5 & 1 \\ 0 & 0 & 0 & 5 & -4 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}.$$

Find the determinant of A if it exists. Show and explain your computations. If the determinant doesn't exist, explain why it doesn't.

answer: The matrix A is a product of two matrices L and U (say), so $\det(A) = \det(L)\det(U)$. But each factor is triangular and so has determinant equal to the product of its diagonal entries. So

$$\begin{aligned} \det A &= ((-4)(-1)(2)(-1)(-3)) \cdot ((4)(-2)(-1)(5)(3)) \\ &= (24) \cdot (120) \\ &= 2880. \end{aligned}$$

5. (1 point) Let

$$A = \begin{pmatrix} 5 & 5 & 3 & 9 \\ 1 & 8 & 0 & 2 \\ 3 & 6 & 3 & 5 \\ 1 & 9 & 0 & 3 \end{pmatrix}.$$

Find the determinant of A if it exists. Show and explain your computations. If the determinant doesn't exist, explain why it doesn't.

answer: The matrix has determinant 51.

Total for assignment: 5 points