Chapter 2

Math 2890-003
Spring 2016 **Name** ______________________
Due Feb 23

1. (1 point) Let

$$A = \begin{pmatrix} -5 & -1 & 9 \\ -8 & -1 & 6 \\ -3 & -3 & -2 \\ -4 & -1 & -6 \end{pmatrix}.$$

   Find A^T, the transpose of A.

   answer: The transpose $A^T = \begin{pmatrix} -5 & -8 & -3 & -4 \\ -1 & -1 & -3 & -1 \\ 9 & 6 & -2 & -6 \end{pmatrix}$.

2. (1 point) Let

$$A = \begin{pmatrix} 0 & 5 & -1 & 4 & -6 \\ 5 & 8 & -2 & 3 & -9 \\ -1 & -2 & -9 & 7 & 3 \\ 4 & 3 & 7 & 4 & -1 \\ -6 & -9 & 3 & -1 & 7 \end{pmatrix}.$$

   Is A symmetric? Explain your answer.

   answer: A is symmetric.

3. (1 point) Let

$$A = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}.$$

Is A orthogonal? Explain your answer.

answer: A is not orthogonal since $I \neq A^T A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$.

4. (3 points) For each of the following matrices, find the inverse if it exists. Show your work.

(a) $A = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 0 & -3 \\ -6 & -6 & -17 \end{pmatrix}.$

answer: $A^{-1} = \begin{pmatrix} -18 & -1 & -3 \\ 1 & 1 & 0 \\ 6 & 0 & 1 \end{pmatrix}.$

(b) $B = \begin{pmatrix} 1 & 4 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}.$

answer: $A^{-1} = \begin{pmatrix} 1 & -4 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}.$

(c) $C = \begin{pmatrix} 3 & -9 & 1 \\ -5 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$

answer: $A^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 5 \\ 1 & 9 & 42 \end{pmatrix}.$

5. (1 point) Let

$$A = \begin{pmatrix} 1 & 4 & 9 \\ 0 & 6 & 4 \\ 7 & 4 & 5 \\ -2 & 4 & 4 \\ -7 & -7 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 8 & 4 & 6 \\ 1 & -3 & 0 \\ 6 & -5 & 5 \\ -4 & 9 & -4 \\ 8 & -3 & -8 \end{pmatrix}.$$

Compute the sum $A + B$ (showing your work) if it is defined; otherwise, explain why it is not defined.

answer: The sum $A + B = \begin{pmatrix} 9 & 8 & 15 \\ 1 & 3 & 4 \\ 13 & -1 & 10 \\ -6 & 13 & 0 \\ 1 & -10 & -9 \end{pmatrix}$.

6. (1 point) Let

$$A = \begin{pmatrix} -6 & -3 & 1 \\ 1 & -1 & -6 \\ 8 & 1 & 9 \\ -7 & -8 & -3 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 7 \\ -3 \\ 6 \end{pmatrix}.$$

Compute the product Av (showing your work) if it is defined; otherwise, explain why it is not defined.

answer: The product $Av = \begin{pmatrix} -27 \\ -26 \\ 107 \\ -43 \end{pmatrix}$.

7. (1 point) Let

$$v = \begin{pmatrix} 4 & -7 & -1 & 2 & 3 \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} 0 & -3 & -9 \\ 2 & -1 & 4 \\ 2 & 5 & -1 \\ 4 & 1 & 2 \\ -6 & 7 & 7 \end{pmatrix}.$$

Compute the product vA (showing your work) if it is defined; otherwise, explain why it is not defined.

answer: The product $vA = \begin{pmatrix} -26 & 13 & -38 \end{pmatrix}$.

8. (1 point) Let

$$A = \begin{pmatrix} 9 & -1 \\ 7 & -2 \\ -3 & -7 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 8 & -3 & 8 \\ -3 & 7 & -6 & 8 \end{pmatrix}.$$

Compute the product AB (showing your work) if it is defined; otherwise, explain why it is not defined.

answer: The product $AB = \begin{pmatrix} -15 & 65 & -21 & 64 \\ -8 & 42 & -9 & 40 \\ 27 & -73 & 51 & -80 \end{pmatrix}$.

9. **(1 point)** Suppose A is a 25×17 matrix and B is a 17×39 matrix. What size is the product AB if it is defined? Explain your answer.

10. **(1 point)** Let

$$A = \begin{pmatrix} 1 & -8 & -4 \\ -3 & 8 & -5 \\ 1 & -4 & 8 \\ 3 & -1 & 5 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -3 & -4 & 1 & 7 \\ 3 & -8 & -6 & 4 \\ 9 & -6 & -4 & -5 \end{pmatrix}.$$

Compute column 4 of the product AB by first writing it as a linear combination of the columns of A. Show your work.

answer: $\begin{pmatrix} 1 \\ -3 \\ 1 \\ 3 \end{pmatrix}(7) + \begin{pmatrix} -8 \\ 8 \\ -4 \\ -1 \end{pmatrix}(4) + \begin{pmatrix} -4 \\ -5 \\ 8 \\ 5 \end{pmatrix}(-5) = \begin{pmatrix} -5 \\ 36 \\ -49 \\ -8 \end{pmatrix}$

11. (1 point) Let

$$A = \begin{pmatrix} 6 & -7 & 2 & -3 \\ 5 & -8 & 3 & -7 \\ 8 & -8 & 5 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 8 & 8 & 0 \\ -2 & -9 & -9 \\ -3 & -5 & 7 \\ -9 & 1 & 7 \end{pmatrix}.$$

Compute row 3 of the product AB by first writing it as a linear combination of the rows of B. Show your work.

answer: $(8)\begin{pmatrix} 8 & 8 & 0 \end{pmatrix} + (-8)\begin{pmatrix} -2 & -9 & -9 \end{pmatrix} + (5)\begin{pmatrix} -3 & -5 & 7 \end{pmatrix} + (-2)\begin{pmatrix} -9 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 83 & 109 & 93 \end{pmatrix}$

12. (1 point) Let

$$A = \begin{pmatrix} -7 & -1 & 0 \\ 5 & 4 & -4 \\ -3 & -6 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 6 & 4 & 9 & 0 & 1 \\ 3 & -9 & 0 & -1 & 4 \\ -5 & 7 & -1 & -9 & 9 \end{pmatrix}.$$

Compute the entry in row 2, column 4 of the product AB. Show your work.

answer: $\begin{pmatrix} 5 & 4 & -4 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \\ -9 \end{pmatrix} = 32.$

13. (1 point) Let

$$A = \begin{pmatrix} -7 & -4 & 6 \\ 9 & -1 & 2 \\ 4 & -4 & 3 \\ 8 & 2 & -9 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 9 & -7 & -5 & 7 \\ -9 & 1 & -6 & 6 \\ -3 & -4 & -7 & -3 \end{pmatrix}.$$

Write the product AB as a sum of 3 rank one matrices. Show your work.

answer: $AB = \begin{pmatrix} -7 \\ 9 \\ 4 \\ 8 \end{pmatrix} \begin{pmatrix} 9 & -7 & -5 & 7 \end{pmatrix} + \begin{pmatrix} -4 \\ -1 \\ -4 \\ 2 \end{pmatrix} \begin{pmatrix} -9 & 1 & -6 & 6 \end{pmatrix} +$

$\begin{pmatrix} 6 \\ 2 \\ 3 \\ -9 \end{pmatrix} \begin{pmatrix} -3 & -4 & -7 & -3 \end{pmatrix}$

$$= \begin{pmatrix} -63 & 49 & 35 & -49 \\ 81 & -63 & -45 & 63 \\ 36 & -28 & -20 & 28 \\ 72 & -56 & -40 & 56 \end{pmatrix} + \begin{pmatrix} 36 & -4 & 24 & -24 \\ 9 & -1 & 6 & -6 \\ 36 & -4 & 24 & -24 \\ -18 & 2 & -12 & 12 \end{pmatrix} + \begin{pmatrix} -18 & -24 & -42 & -18 \\ -6 & -8 & -14 & -6 \\ -9 & -12 & -21 & -9 \\ 27 & 36 & 63 & 27 \end{pmatrix}$$

14. (1 point) Let

$$A = \begin{pmatrix} -4 & 3 & 4 & 0 & 9 & -6 \\ -4 & -1 & 20 & 16 & -3 & -14 \\ 3 & 3 & -24 & -21 & 9 & 15 \\ 1 & -1 & 0 & 1 & -3 & 1 \\ 4 & 5 & -36 & -32 & 15 & 22 \end{pmatrix}.$$

Find the rank of the matrix A. Explain your answer.

answer: The rank of A is 2

15. (1 point) Let

$$A = \begin{pmatrix} 2 & 4 & 0 & -2 \\ 10 & 22 & -5 & -14 \\ 8 & 8 & 15 & 6 \\ 6 & 4 & 20 & 6 \end{pmatrix} \quad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 4 & -4 & 1 & 0 \\ 3 & -4 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 2 & 4 & 0 & -2 \\ 0 & 2 & -5 & -4 \\ 0 & 0 & -5 & -2 \\ 0 & 0 & 0 & -4 \end{pmatrix} \text{ and } b = \begin{pmatrix} -8 \\ -60 \\ 24 \\ 88 \end{pmatrix}.$$

Use the LU factorization $A = LU$ to solve the matrix equation $Ax = b$. Show your work.

answer: The vector $y = \begin{pmatrix} -8 \\ -20 \\ -24 \\ 32 \end{pmatrix}$ is the solution of the system $Ly = b$.

and the vector $x = \begin{pmatrix} 0 \\ -6 \\ 8 \\ -8 \end{pmatrix}$ is the solution of $Ux = y$ and so of $Ax = b$.

16. (1 point) Let

$$A = \begin{pmatrix} 3 & -2 & 0 & 2 \\ 15 & -11 & 3 & 15 \\ 9 & -8 & 1 & 11 \\ 15 & -10 & 10 & 15 \end{pmatrix}.$$

Use Gaussian Elimination (or Wedderburn rank reduction) to find the LU factorization of the matrix A. Show your work.

answer: $A = LU$ where $L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 5 & 0 & -2 & 1 \end{pmatrix}$ and $U = \begin{pmatrix} 3 & -2 & 0 & 2 \\ 0 & -1 & 3 & 5 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & 0 & -5 \end{pmatrix}$

17. (1 point) Let

$$A = \begin{pmatrix} 6 & 6 & 6 & -25 \\ 2 & 9 & -3 & -16 \\ 10 & 0 & 15 & -15 \\ 4 & -12 & -6 & -2 \end{pmatrix}.$$

Use Gaussian Elimination with Partial Pivoting (or Wedderburn rank reduction) to find a permuted LU (or permuted LDU) factorization of the matrix A. Show your work.

answer: Either $A = LDU$ where $L = \begin{pmatrix} 6 & 6 & -9 & -8 \\ 2 & 9 & -15 & 0 \\ 10 & 0 & 0 & 0 \\ 4 & -12 & 0 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 1/10 & 0 & 0 & 0 \\ 0 & -1/12 & 0 & 0 \\ 0 & 0 & -1/15 & 0 \\ 0 & 0 & 0 & -1/8 \end{pmatrix}$ and $U = \begin{pmatrix} 10 & 0 & 15 & -15 \\ 0 & -12 & -12 & 4 \\ 0 & 0 & -15 & -10 \\ 0 & 0 & 0 & -8 \end{pmatrix}$,

or else $A = LU$ with the same U, but with $L = \begin{pmatrix} 0.6000000000000001 & -0.5 & 0.6 & 1 \\ 0.2 & -0.75 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0.4 & 1 & 0 & 0 \end{pmatrix}$

18. (2 points) Let

$$A = \begin{pmatrix} 1 & -1 & 5 & 0 & -18 \\ -4 & 4 & -19 & -5 & 59 \\ -4 & 4 & -20 & 1 & 74 \\ -1 & 1 & -8 & 12 & 51 \\ 1 & -1 & 4 & 10 & 5 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} -2 \\ 5 \\ 10 \\ 10 \\ 32 \end{pmatrix}.$$

(a) Is v in the column space of A? Show your work.

(b) Is v in the null space of A? Show your work.

answer: (a) Since $(A|v)$ has a pivot in the last column $Ax = v$ is not consistent, and v is **not** in the column space of A.

(b) v is **not** in the null space of A since $Av \neq 0$.

19. (2 points) Let

$$A = \begin{pmatrix} 1 & 4 & -4 & -6 & 13 \\ 2 & 0 & 0 & 4 & -6 \\ 2 & -3 & 3 & 10 & -18 \\ -1 & -1 & 1 & 0 & -1 \\ 2 & 2 & -2 & 0 & 2 \end{pmatrix}.$$

(a) Find a nonzero vector in the column space of A. Show your work.

(b) Find a nonzero vector in the null space of A. Show your work.

answer: (a) I found the vector $\begin{pmatrix} 47 \\ -10 \\ -49 \\ -8 \\ 16 \end{pmatrix}$ in the column space. You'll probably find a different vector.

(b) I found the vector $\begin{pmatrix} -5 \\ 10 \\ 0 \\ -5 \\ -5 \end{pmatrix}$ in the null space. You'll probably find a different vector.

20. (1 point) Let

$$A = \begin{pmatrix} -5 & 5 & 2 & 25 & 18 & -2 \\ -2 & 0 & 2 & 10 & 2 & -1 \\ -2 & -4 & 4 & 8 & -8 & -4 \\ -5 & -3 & 5 & 16 & -1 & 0 \\ 2 & 2 & 0 & 6 & 0 & -2 \end{pmatrix}.$$

Find bases for Col(A) and Nul(A).

Hint: The reduced row echelon form of A is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & 5 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

answer: The columns of the matrix $\begin{pmatrix} -5 & 5 & 2 & -2 \\ -2 & 0 & 2 & -1 \\ -2 & -4 & 4 & -4 \\ -5 & -3 & 5 & 0 \\ 2 & 2 & 0 & -2 \end{pmatrix}$ are a basis for the column space of A.

The columns of the matrix $\begin{pmatrix} 0 & 2 \\ -3 & -2 \\ -5 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ are a basis for the null space of A.

21. (1 point) Let

$$A = \begin{pmatrix} 58 & -15 & -144 & -204 & 11 \\ 101 & -27 & -249 & -357 & -4 \\ -45 & 12 & 111 & 159 & 1 \\ 15 & -4 & -37 & -53 & 0 \end{pmatrix}.$$

Find bases for Col(A), Nul(A), Col(A^T) and Nul(A^T).

Hint: If you form the matrix $(A|I)$ and use row operations to put the A part in reduced row echelon form you get $(R|S)$ where

$$R = \begin{pmatrix} 1 & 0 & -3 & -3 & 0 \\ 0 & 1 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \ S = \begin{pmatrix} 1 & 3 & 1 & -21 \\ 4 & 13 & 8 & -79 \\ -1 & -7 & -16 & 3 \\ 1 & 7 & 17 & 0 \end{pmatrix}.$$

answer: The columns of the matrix $\begin{pmatrix} 58 & -15 & 11 \\ 101 & -27 & -4 \\ -45 & 12 & 1 \\ 15 & -4 & 0 \end{pmatrix}$ form a basis for the column space of A.

The columns of the matrix $\begin{pmatrix} 3 & 3 \\ 2 & -2 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ form a basis for the null space of A.

The columns of the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & -2 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ form a basis for the column space of A^T.

The columns of the matrix $\begin{pmatrix} 1 \\ 7 \\ 17 \\ 0 \end{pmatrix}$ form a basis for the null space of A^T.

Total for assignment: 25 points