Chapter 1

Math 2890-003
Spring 2016 **Name** _____
Due Feb 02

1. (1 point) Write out the augmented matrix corresponding to the linear system.

$$\begin{array}{rcrcrcrcrcrcr}
4x_1 & + & 5x_2 & - & 3x_3 & - & 3x_4 & + & x_5 & + & 7x_6 & = & -2 \\
-7x_1 & + & 2x_2 & + & 9x_3 & + & 8x_4 & & & + & 3x_6 & = & 8 \\
 & - & 8x_2 & - & 2x_3 & + & 6x_4 & - & 2x_5 & - & 3x_6 & = & 9 \\
x_1 & - & 3x_2 & & & - & 5x_4 & + & 8x_5 & + & 2x_6 & = & 0 \\
3x_1 & + & x_2 & - & 3x_3 & + & 5x_4 & + & 2x_5 & + & x_6 & = & 1
\end{array}$$

2. (1 point) Write out the linear system corresponding to the augmented matrix.

$$\left(\begin{array}{cccccc|c}
1 & 8 & -2 & 7 & 9 & 0 & 2 \\
3 & -7 & 8 & 2 & 0 & 2 & 6 \\
0 & 0 & 0 & 1 & -2 & 2 & 3 \\
-4 & 2 & -1 & 3 & 8 & 1 & 5 \\
5 & 9 & 5 & 4 & 1 & -9 & -4
\end{array}\right)$$

3. (1 point) Let

$$u = \begin{pmatrix} 1 \\ -5 \\ 5 \end{pmatrix}, \; v = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix}, \; w = \begin{pmatrix} 5 \\ -17 \\ 29 \end{pmatrix} \text{ and } x = \begin{pmatrix} -1 \\ -7 \\ -11 \end{pmatrix}.$$

Do the given vectors span $\mathbb{R}^3$? Show your work. Explain your answer.

answer: The vectors do **not** span $\mathbb{R}^3$ since (after constructing a matrix using the vectors as the columns) there's no pivot in *row* 3.

4. (1 point) Let

$$u = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}, \; v = \begin{pmatrix} -6 \\ -2 \\ 18 \end{pmatrix} \text{ and } w = \begin{pmatrix} 6 \\ -8 \\ 27 \end{pmatrix}.$$

Are the given vectors linearly independent? Show your work. Explain your answer.

answer: The vectors are **not** linearly independent since (after constructing a matrix using the vectors as the columns) there is no pivot in *column* 3.

5. (2 points) Determine whether the following matrices are

RREF = in reduced row echelon form,

UREF = in row echelon form, but not in reduced row echelon form, or

NOEF = neither in row echelon form or in reduced row echelon form.

(a)
$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(d)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

answer:

(a) UREF

(b) RREF

(c) RREF

(d) NOEF

6. (1 point) Let

$$A = \begin{pmatrix} 3 & 12 & 3 & 11 \\ -6 & -24 & -11 & 1 \\ 9 & 36 & 4 & 64 \end{pmatrix}.$$

Use Gaussian elimination to reduce the matrix A to row echelon form. Show your work.

answer:

$$\begin{pmatrix} 3 & 12 & 3 & 11 \\ -6 & -24 & -11 & 1 \\ 9 & 36 & 4 & 64 \end{pmatrix} \sim \begin{pmatrix} 3 & 12 & 3 & 11 \\ 0 & 0 & -5 & 23 \\ 9 & 36 & 4 & 64 \end{pmatrix}$$

$$\sim \begin{pmatrix} 3 & 12 & 3 & 11 \\ 0 & 0 & -5 & 23 \\ 0 & 0 & -5 & 31 \end{pmatrix}$$

$$\sim \begin{pmatrix} 3 & 12 & 3 & 11 \\ 0 & 0 & -5 & 23 \\ 0 & 0 & 0 & 8 \end{pmatrix}$$

7. (1 point) Let

$$A = \begin{pmatrix} 6 & -1 & -14 & -40 \\ -12 & 10 & 27 & 27 \\ 18 & -18 & -12 & -30 \end{pmatrix}.$$

Use Gaussian elimination with partial pivoting to reduce the matrix A to row echelon form. Show your work.

answer: Gaussian elimination with partial pivoting reduces A to

$$\begin{pmatrix} 18 & -18 & -12 & -30 \\ 0 & 5 & -10 & -30 \\ 0 & 0 & 15 & -5 \end{pmatrix}.$$

Here $A = LU$ with U as above and $L = \begin{pmatrix} 0.3333 & 1 & 0 \\ -0.6667 & -0.4 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

8. (1 point) Let

$$A = \begin{pmatrix} 4 & 3 & -14 & -7 & -2 \\ 3 & -2 & -2 & -1 & 4 \\ -3 & 1 & 4 & 2 & -2 \\ 5 & 2 & -14 & -7 & -3 \end{pmatrix}.$$

Find the reduced row echelon form of A. Show your work.

answer: The reduced row echelon form is $\begin{pmatrix} 1 & 0 & -2 & -1 & 0 \\ 0 & 1 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

9. (1 point) Let

$$A = \begin{pmatrix} 9 & -7 & -7 & -4 \\ -1 & 4 & -3 & 6 \\ -4 & 3 & 3 & 3 \\ 7 & -6 & -7 & -6 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} -5 \\ 33 \\ 3 \\ -7 \end{pmatrix}.$$

Solve the equation $Ax = b$ (showing your work) or explain why it doesn't have a solution.

answer: The solution is $x = \begin{pmatrix} 3 \\ 6 \\ -2 \\ 1 \end{pmatrix}$.

10. (1 point) Let

$$A = \begin{pmatrix} -2 & 0 & 1 \\ -2 & -2 & -3 \\ 1 & 3 & -1 \\ -1 & -3 & 2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} -3 \\ 1 \\ -1 \\ -2 \end{pmatrix}.$$

Solve the equation $Ax = b$ (showing your work) or explain why it doesn't have a solution.

answer: The system is inconsistent because the augmented matrix $(A|b)$ has a pivot in the last column.

11. (1 point) Let

$$A = \begin{pmatrix} -1 & 1 & 3 \\ -1 & -1 & -1 \\ 1 & -4 & 1 \\ 3 & -4 & -1 \\ 1 & -3 & 2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 12 \\ -8 \\ 8 \\ 4 \\ 13 \end{pmatrix}.$$

Solve the equation $Ax = b$ (showing your work) or explain why it doesn't have a solution.

answer: The solution is $x = \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix}$.

12. (1 point) Let

$$A = \begin{pmatrix} 5 & 5 & 20 & 30 & 0 & 1 \\ 2 & -2 & 12 & 8 & 4 & -2 \\ -5 & 4 & -29 & -21 & 4 & 0 \\ -2 & -4 & -6 & -14 & -3 & -2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} -75 \\ -30 \\ -29 \\ 70 \end{pmatrix}.$$

Find the general solution of the equation $Ax = b$. Show your work.

HINT: The augmented matrix $(A|b)$ has reduced row echelon form

$$\left(\begin{array}{cccccc|c} 1 & 0 & 5 & 5 & 0 & 0 & -7 \\ 0 & 1 & -1 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 0 & 1 & 0 & -8 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

answer: The general solution is $x = \begin{pmatrix} -7 \\ -8 \\ 0 \\ 0 \\ -8 \\ 0 \end{pmatrix} + \begin{pmatrix} -5 & -5 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} y$, where $y \in R^2$.

13. (1 point) Let

$$v = \begin{pmatrix} 5 \\ -4 \\ 8 \\ -1 \\ 0 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} -7 \\ -7 \\ -3 \\ -2 \\ 2 \end{pmatrix}.$$

Compute the sum $v + w$ if it is defined; otherwise, explain why it is not defined.

answer: The sum $v + w = \begin{pmatrix} -2 \\ -11 \\ 5 \\ -3 \\ 2 \end{pmatrix}$.

14. (1 point) Let

$$v = \begin{pmatrix} -6 \\ 9 \\ -1 \\ 2 \\ -4 \\ -7 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} 2 \\ 2 \\ -8 \\ 0 \\ 7 \end{pmatrix}.$$

Compute the sum $v + w$ if it is defined; otherwise, explain why it is not defined.

answer: The sum $v + w$ is not defined because v and w have different dimensions.

15. (1 point) Let

$$\alpha = -7, \quad \beta = 5, \quad v = \begin{pmatrix} 2 \\ 9 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} -1 \\ 7 \\ 6 \\ -1 \end{pmatrix}.$$

Compute the linear combination $v\alpha + w\beta$, or explain why it is impossible. Show your work.

answer: The linear combination $v\alpha + w\beta = \begin{pmatrix} -19 \\ -28 \\ 37 \\ -54 \end{pmatrix}$.

Total for assignment: 16 points