Matrix Factorizations

We've reached the end of the course. We now have six factorizations. Three are related to linear systems while three are related to eigensystems.

Linear Systems: $Ax = b$

Given matrix $A_{m\times n}$ and vector b find the vector x that makes Ax as close to b as possible.

- A is invertible (most square matrices, i.e., $m = n$).

  $A = LDU$

  L, U triangular, D diagonal (all $m \times m = n \times n$)
- A has linearly independent columns (most matrices with $m \geq n$).

  $A = QDR$

  $R_{n\times n}$ triangular, $D_{n\times n}$ diagonal, $Q_{m\times n}$ with $Q^T QD = I_{n\times n}$
- A is any matrix.

  $A = U\Sigma V^*$

  $U_{m\times m}, V_{n\times n}$ unitary, $\Sigma_{m\times n}$ diagonal with non-negative entries

Eigen Systems: $Ax = x\lambda$

Given square matrix $A_{m\times m}$ find scalars λ and vectors $x \neq 0$ that satisfy equation.

- A is normal ($AA^* = A^*A$).

  $A = UDU^*$

  U unitary, D diagonal
- A is diagonalizable (most square matrices).

  $A = PDP^{-1}$

  P invertible, D diagonal
- A is any square matrix.

  $A = UTU^*$

  U unitary, T triangular

What you are responsible for:

- Knowing how to find each of these six factorizations when you are given a matrix.
- For the three factorizations related to linear systems, knowing how to use the factorization to find the solution (or least-squares solution) to a linear system.
- For the three factorizations related to eigensystems, knowing how to use the factorization to find the eigenvalues and eigenvectors of the matrix.