

Gates

For this exam you may use the gates on this page along with their controlled versions.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/8} \end{bmatrix}$$

$$F = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \quad K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\theta) \equiv e^{-i(\theta/2)X} = \cos(\theta/2)I - i \sin(\theta/2)X = \begin{bmatrix} \cos \theta/2 & -i \sin \theta/2 \\ -i \sin \theta/2 & \cos \theta/2 \end{bmatrix}$$

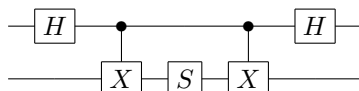
$$R_y(\theta) \equiv e^{-i(\theta/2)Y} = \cos(\theta/2)I - i \sin(\theta/2)Y = \begin{bmatrix} \cos \theta/2 & -\sin \theta/2 \\ \sin \theta/2 & \cos \theta/2 \end{bmatrix}$$

$$R_z(\theta) \equiv e^{-i(\theta/2)Z} = \cos(\theta/2)I - i \sin(\theta/2)Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

Please show and explain your work.

The first five problems are all worth the same amount.

1. (20 points) Find the unitary matrix that corresponds to the Quantum circuit.

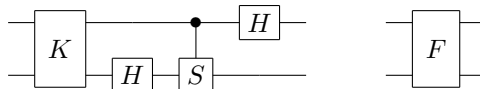


2. (20 points) Find the matrices $K \otimes X$ and $X \otimes K$.

3. (20 points) Draw a Quantum circuit that corresponds to the following unitary matrix. Use only the gates on the other sheet along with (possibly) controlled versions of those gates. Show that your circuit works.

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

4. (20 points) Determine whether or not the following two circuits are equivalent. Prove your result.



5. (20 points) The *Fredkin gate* is a gate on 3 qubits that swaps 101 and 110 while leaving the six other strings fixed. Draw a Quantum circuit for the Fredkin gate that uses only the gates on the other sheet along with (possibly) controlled versions of those gates. Show that your circuit works.

6. (extra credit) Find $A \otimes B$ where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$