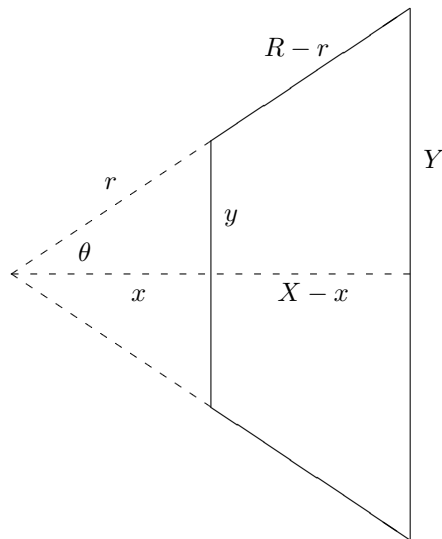


### Surface Area of the Frustum of a Cone

The following picture represents the frustum of a cone with the vertex on the left and the base on the right. There are two cones in the picture. The smaller cone (shown with dashed lines) has height  $x$  and a base with radius  $y$ . The larger cone has height  $X$  and a base with radius  $Y$ .



Using the formula for the surface area of a cone that we derived last time the surface area of the frustum (the area of the larger cone minus the area of the smaller cone) is

$$\begin{aligned}
 \pi RY - \pi r y &= \pi \sec(\theta) X \tan(\theta) X - \pi \sec(\theta) x \tan(\theta) x \\
 &= \pi \sec(\theta) \tan(\theta) (X^2 - x^2) \\
 &= \pi \sec(\theta) (X - x) \tan(\theta) (X + x) \\
 &= \pi (R - r) (Y + y) \\
 &= 2\pi (R - r) \left( \frac{Y + y}{2} \right)
 \end{aligned}$$

### Frustum of Infinitesimal Width

Write  $R = r + dr$  and  $Y = y + dy$  where  $dr$  and  $dy$  are infinitesimal. We have the surface area of the frustum

$$\begin{aligned}
 2\pi dr \left( y + \frac{dy}{2} \right) &= 2\pi y dr + \pi dr dy \\
 &\approx 2\pi y dr
 \end{aligned}$$