

Chapter 4

Math 2890-001

Fall 2017

Due Nov 13

Name _____

1. (1 point) Let

$$A = \begin{pmatrix} -154 & 112 & 58 & -36 \\ -183 & 133 & 68 & -42 \\ -198 & 144 & 77 & -48 \\ -226 & 164 & 86 & -53 \end{pmatrix}.$$

Compute A^9 . Show and explain your work.

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & 4 & 2 & 2 \\ 1 & 5 & 4 & 1 \\ 2 & 4 & -3 & 9 \\ 2 & 5 & -1 & 9 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad P^{-1} = \begin{pmatrix} -43 & 32 & 18 & -12 \\ 14 & -10 & -5 & 3 \\ -7 & 5 & 2 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

answer:

$$A^9 = PD^9P^{-1}$$

$$\begin{aligned} &= \begin{pmatrix} 1 & 4 & 2 & 2 \\ 1 & 5 & 4 & 1 \\ 2 & 4 & -3 & 9 \\ 2 & 5 & -1 & 9 \end{pmatrix} \begin{pmatrix} 512 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -43 & 32 & 18 & -12 \\ 14 & -10 & -5 & 3 \\ -7 & 5 & 2 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -22084 & 16432 & 9238 & -6156 \\ -22113 & 16453 & 9248 & -6162 \\ -44058 & 32784 & 18437 & -12288 \\ -44086 & 32804 & 18446 & -12293 \end{pmatrix}. \end{aligned}$$

2. (1 point) Let

$$A = \begin{pmatrix} 0.6 & 0.2 & 0.5 & 0.1 \\ 0.1 & 0.6 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.3 & 0.1 \\ 0.2 & 0.1 & 0.1 & 0.5 \end{pmatrix}.$$

Find a steady state probability vector for the stochastic matrix A . Show and explain your work.

answer: A steady state probability vector x has to be a solution of

$$(A - I)x = 0$$

and have nonnegative entries that sum to 1. Since

$$A - I = \begin{pmatrix} -0.4 & 0.2 & 0.5 & 0.1 \\ 0.1 & -0.4 & 0.1 & 0.3 \\ 0.1 & 0.1 & -0.7 & 0.1 \\ 0.2 & 0.1 & 0.1 & -0.5 \end{pmatrix} \text{ we get } x = \begin{pmatrix} 0.3581 \\ 0.2905 \\ 0.125 \\ 0.2264 \end{pmatrix}$$

after row reducing and making an appropriate choice of free variable.

Total for assignment: 2 points