Chapter 1

Math 2890-001
Fall 2017 **Name** ____________________________________
Due Sep 22

1. (1 point) Write out the augmented matrix corresponding to the linear system.

$$\begin{array}{rcrcrcrcrcrcr} 4x_1 & + & 5x_2 & - & 3x_3 & - & 3x_4 & + & x_5 & + & 7x_6 & = & -2 \\ -7x_1 & + & 2x_2 & + & 9x_3 & + & 8x_4 & & & + & 3x_6 & = & 8 \\ & - & 8x_2 & - & 2x_3 & + & 6x_4 & - & 2x_5 & - & 3x_6 & = & 9 \\ x_1 & - & 3x_2 & & & - & 5x_4 & + & 8x_5 & + & 2x_6 & = & 0 \\ 3x_1 & + & x_2 & - & 3x_3 & + & 5x_4 & + & 2x_5 & + & x_6 & = & 1 \end{array}$$

2. (1 point) Write out the linear system corresponding to the augmented matrix.

$$\left(\begin{array}{rrrrrr|r} 1 & 8 & -2 & 7 & 9 & 0 & 2 \\ 3 & -7 & 8 & 2 & 0 & 2 & 6 \\ 0 & 0 & 0 & 1 & -2 & 2 & 3 \\ -4 & 2 & -1 & 3 & 8 & 1 & 5 \\ 5 & 9 & 5 & 4 & 1 & -9 & -4 \end{array}\right)$$

3. (1 point) Let

$$u = \begin{pmatrix} 3 \\ -3 \\ 4 \end{pmatrix}, \ v = \begin{pmatrix} 6 \\ -2 \\ 6 \end{pmatrix}, \ w = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } x = \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix}.$$

Do the given vectors span $\mathbb{R}^3$?
Show your work. Explain your answer.

answer: The vectors span $\mathbb{R}^3$ since (after constructing a matrix using the vectors as the columns) every row has a pivot.

4. (1 point) Let

$$u = \begin{pmatrix} -2 \\ 0 \\ 3 \\ 2 \end{pmatrix}, \; v = \begin{pmatrix} -6 \\ -2 \\ 18 \\ 8 \end{pmatrix} \text{ and } w = \begin{pmatrix} 6 \\ -8 \\ 0 \\ 2 \end{pmatrix}.$$

Are the given vectors linearly independent?
Show your work. Explain your answer.

answer: Yes, the vectors are linearly independent since (after constructing a matrix using the vectors as the columns) there is a pivot in every column.

5. (2 points) Determine whether the following matrices are

RREF = in reduced row echelon form,

UREF = in row echelon form, but not in reduced row echelon form, or

NOEF = neither in row echelon form or in reduced row echelon form.

(a)
$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(d)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

answer:

(a) RREF
(b) UREF
(c) RREF
(d) NOEF

6. (1 point) Let

$$A = \begin{pmatrix} 3 & 6 & 3 & 1 \\ -6 & -12 & -11 & 1 \\ 9 & 18 & 4 & 8 \end{pmatrix}.$$

Use **Gaussian elimination** to reduce the matrix A to row echelon form. Notice that this problem doesn't involve a linear system, it is just an exercise in *row reduction*.

Show your work.

answer:

$$\begin{pmatrix} 3 & 6 & 3 & 1 \\ -6 & -12 & -11 & 1 \\ 9 & 18 & 4 & 8 \end{pmatrix} \sim \begin{pmatrix} 3 & 6 & 3 & 1 \\ 0 & 0 & -5 & 3 \\ 9 & 18 & 4 & 8 \end{pmatrix}$$

$$\sim \begin{pmatrix} 3 & 6 & 3 & 1 \\ 0 & 0 & -5 & 3 \\ 0 & 0 & -5 & 5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 3 & 6 & 3 & 1 \\ 0 & 0 & -5 & 3 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

7. (1 point) Let

$$A = \begin{pmatrix} 9 & 4 & -8 & -2 \\ 18 & 12 & -24 & 12 \\ -3 & 4 & 1 & 7 \end{pmatrix}.$$

Use **Gaussian elimination with partial pivoting** to reduce the matrix A to row echelon form. Notice that this problem doesn't involve a linear system, it is just an exercise in *row reduction*.

Show your work.

answer:

$$\begin{pmatrix} 9 & 4 & -8 & -2 \\ 18 & 12 & -24 & 12 \\ -3 & 4 & 1 & 7 \end{pmatrix} \sim \begin{pmatrix} 18 & 12 & -24 & 12 \\ 9 & 4 & -8 & -2 \\ -3 & 4 & 1 & 7 \end{pmatrix}$$

$$\sim \begin{pmatrix} 18 & 12 & -24 & 12 \\ 0 & -2 & 4 & -8 \\ -3 & 4 & 1 & 7 \end{pmatrix}$$

$$\sim \begin{pmatrix} 18 & 12 & -24 & 12 \\ 0 & -2 & 4 & -8 \\ 0 & 6 & -3 & 9 \end{pmatrix}$$

$$\sim \begin{pmatrix} 18 & 12 & -24 & 12 \\ 0 & 6 & -3 & 9 \\ 0 & -2 & 4 & -8 \end{pmatrix}$$

$$\sim \begin{pmatrix} 18 & 12 & -24 & 12 \\ 0 & 6 & -3 & 9 \\ 0 & 0 & 3 & -5 \end{pmatrix}$$

8. (1 point) Let

$$A = \begin{pmatrix} 4 & 3 & -14 & -7 & -2 \\ 3 & -2 & -2 & -1 & 4 \\ 1 & 4 & -10 & -5 & -4 \\ 5 & 2 & -14 & -7 & -3 \end{pmatrix}.$$

Find the reduced row echelon form of A. Notice that this problem doesn't involve a linear system, it is just an exercise in *row reduction.* Also, you are not obliged to use Gaussian elimination for this problem; some semblance of free will is returned.

Show your work.

answer: The reduced row echelon form is $\begin{pmatrix} 1 & 0 & -2 & -1 & 0 \\ 0 & 1 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

9. (1 point) Let

$$A = \begin{pmatrix} -3 & 3 & -2 \\ -9 & 12 & -3 \\ 6 & -15 & -6 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 3 \\ 3 \\ 15 \end{pmatrix}.$$

Solve the equation $Ax = b$ or explain why it doesn't have a solution. Show your work.

answer:

$$(A \mid b) = \left(\begin{array}{ccc|c} -3 & 3 & -2 & 3 \\ -9 & 12 & -3 & 3 \\ 6 & -15 & -6 & 15 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} -3 & 3 & -2 & 3 \\ 0 & 3 & 3 & -6 \\ 0 & 0 & -1 & 3 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right)$$

The solution is $x = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.

10. (1 point) Let

$$A = \begin{pmatrix} 2 & 3 & 1 \\ -2 & 1 & 3 \\ 4 & 10 & 6 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}.$$

Solve the equation $Ax = b$ or explain why it doesn't have a solution. Show your work.

answer: The system is inconsistent because the augmented matrix $(A \mid b)$ has a pivot in the last column.

11. (1 point) Let

$$A = \begin{pmatrix} -1 & 1 & 3 \\ -1 & -1 & -1 \\ 1 & -4 & 1 \\ 1 & -3 & 2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 10 \\ -8 \\ 3 \\ 8 \end{pmatrix}.$$

Solve the equation $Ax = b$ or explain why it doesn't have a solution. Show your work.

answer:

$$(A\,|\,b) = \left(\begin{array}{rrr|r} -1 & 1 & 3 & 10 \\ -1 & -1 & -1 & -8 \\ 1 & -4 & 1 & 3 \\ 1 & -3 & 2 & 8 \end{array}\right)$$

$$\sim \left(\begin{array}{rrr|r} -1 & 1 & 3 & 10 \\ 0 & -2 & -4 & -18 \\ 0 & -3 & 4 & 13 \\ 0 & -2 & 5 & 18 \end{array}\right)$$

$$\sim \left(\begin{array}{rrr|r} -1 & 1 & 3 & 10 \\ 0 & -2 & -4 & -18 \\ 0 & 0 & 10 & 40 \\ 0 & 0 & 9 & 36 \end{array}\right)$$

$$\sim \left(\begin{array}{rrr|r} -1 & 1 & 3 & 10 \\ 0 & -2 & -4 & -18 \\ 0 & 0 & 10 & 40 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \text{so consistent}$$

$$\sim \left(\begin{array}{rrr|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The solution is $x = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$.

12. (1 point) Let

$$A = \begin{pmatrix} 5 & 5 & 20 & 30 & 0 & 1 \\ 2 & -2 & 12 & 8 & 4 & -2 \\ -5 & 4 & -29 & -21 & 4 & 0 \\ -2 & -4 & -6 & -14 & -3 & -2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} -75 \\ -30 \\ -29 \\ 70 \end{pmatrix}.$$

Find the general solution of the equation $Ax = b$. Show your work.

HINT: The augmented matrix $(A \,|\, b)$ has reduced row echelon form

$$\left(\begin{array}{cccccc|c} 1 & 0 & 5 & 5 & 0 & 0 & -7 \\ 0 & 1 & -1 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 0 & 1 & 0 & -8 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

answer: The general solution is

$$x = \begin{pmatrix} -7 \\ -8 \\ 0 \\ 0 \\ -8 \\ 0 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} r \begin{pmatrix} -5 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} s$$

where $r, s \in \mathbb{R}$.

13. (1 point) Let

$$v = \begin{pmatrix} 5 \\ -4 \\ 8 \\ -1 \\ 0 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} -7 \\ -7 \\ -3 \\ -2 \\ 2 \end{pmatrix}.$$

Compute the sum $v + w$ if it is defined; otherwise, explain why it is not defined.

answer: The sum $v + w = \begin{pmatrix} -2 \\ -11 \\ 5 \\ -3 \\ 2 \end{pmatrix}$.

14. (1 point) Let

$$v = \begin{pmatrix} -6 \\ 9 \\ -1 \\ 2 \\ -4 \\ -7 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} 2 \\ 2 \\ -8 \\ 0 \\ 7 \end{pmatrix}.$$

Compute the sum $v + w$ if it is defined; otherwise, explain why it is not defined.

answer: The sum $v + w$ is not defined because v and w have different dimensions.

15. (1 point) Let

$$\alpha = -7, \quad \beta = 5, \quad v = \begin{pmatrix} 2 \\ 9 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} -1 \\ 7 \\ 6 \\ -1 \end{pmatrix}.$$

Compute the linear combination $v\alpha + w\beta$, or explain why it is impossible. Show your work.

answer: The linear combination $v\alpha + w\beta = \begin{pmatrix} -19 \\ -28 \\ 37 \\ -54 \end{pmatrix}$.