

Chapter 5

Math 2890-003

Fall 2016

Due Dec 01

Name _____

1. (1 point) Let

$$A = \begin{pmatrix} 0 & 48 & -38 & -8 \\ 12 & -43 & 24 & 12 \\ 14 & -48 & 25 & 14 \\ 19 & -120 & 82 & 27 \end{pmatrix} \quad \text{and} \quad \lambda = 8.$$

Find an eigenvector for the matrix A that corresponds to the given eigenvalue λ . Show and explain your work.

2. (1 point) Let

$$A = \begin{pmatrix} 72 & -5 & 5 & -28 \\ -20 & -9 & -7 & -4 \\ -196 & 20 & -4 & 98 \\ 182 & -10 & 17 & -61 \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} 1 \\ 0 \\ -4 \\ 2 \end{pmatrix}.$$

Find the eigenvalue for the matrix A that corresponds to the given eigenvector x . Show and explain your work.

3. (1 point) Let

$$A = \begin{pmatrix} -17 & 7 \\ -42 & 18 \end{pmatrix}.$$

Find the eigenvalues (including multiplicities) of A . Show and explain your work.

4. (1 point) Let

$$A = \begin{pmatrix} -1 & 6 & 18 \\ 0 & -4 & -21 \\ 0 & 0 & 3 \end{pmatrix}.$$

Find the eigenvalues (including multiplicities) of A . Show and explain your work.

5. (1 point) Let

$$A = \begin{pmatrix} 13 & -9 & 0 & 0 \\ 18 & -14 & 0 & 0 \\ 10 & -6 & -8 & -2 \\ -2 & 6 & 12 & 2 \end{pmatrix}.$$

Find the eigenvalues (including multiplicities) of A . Show and explain your work.

6. (1 point) Let

$$A = \begin{pmatrix} -3 & 0 & 0 \\ 6 & -5 & 0 \\ -12 & 4 & -1 \end{pmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

7. (1 point) Let

$$A = \begin{pmatrix} 5 & 3 & 6 & -24 \\ 0 & 4 & 0 & -18 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 0 & -2 \end{pmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

8. (1 point) Let

$$A = \begin{pmatrix} -3 & 5 & -3 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{pmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

9. (1 point) Let

$$A = \begin{pmatrix} 1 & 1 & 19 & 9 \\ -2 & -2 & -31 & -13 \\ 0 & 0 & -7 & -2 \\ 0 & 0 & 12 & 3 \end{pmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

10. (1 point) Let

$$x_1 = \begin{pmatrix} 9 \\ -2 \\ 5 \\ -1 \end{pmatrix} \quad x_2 = \begin{pmatrix} -4 \\ -6 \\ 1 \\ -5 \end{pmatrix} \quad x_3 = \begin{pmatrix} -6 \\ -1 \\ 8 \\ 7 \end{pmatrix} \quad x_4 = \begin{pmatrix} -1 \\ 8 \\ 0 \\ 3 \end{pmatrix}$$

and

$$\lambda_1 = -5, \lambda_2 = 1, \lambda_3 = -8, \lambda_4 = 7.$$

Write down a matrix A that has the given vectors as eigenvectors with the corresponding scalars as the eigenvalues.

11. (1 point) Consider the matrix

$$A = \begin{pmatrix} 3109 & 727 & -310 & 114 \\ -9546 & -2238 & 946 & -366 \\ 8826 & 2053 & -891 & 294 \\ 240 & 64 & -16 & 31 \end{pmatrix}.$$

Find all the eigenvalues of the matrix A , and for each eigenvalue find a full complement of linearly independent eigenvectors.

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & 7 & -6 & -9 \\ -3 & -22 & 14 & 34 \\ 3 & 19 & -25 & -14 \\ 0 & 1 & 6 & -10 \end{pmatrix} \quad D = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 7 \end{pmatrix}$$

$$P^{-1} = \begin{pmatrix} -158 & -31 & 22 & 6 \\ -30 & -8 & 2 & -3 \\ -30 & -7 & 3 & -1 \\ -21 & -5 & 2 & -1 \end{pmatrix}.$$

12. (1 point) Let

$$A = \begin{pmatrix} 0.2 & -10.8 & 2.7 \\ 1.1 & 13.7 & -3.1 \\ 4.4 & 50.4 & -11.3 \end{pmatrix}.$$

Is the origin an attractor, repeller or saddle point for the discrete dynamical system $x_{k+1} = Ax_k$? How do you know?

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 1 & -1 \\ -4 & 4 & -3 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 1.3 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}.$$

13. (1 point) Let

$$A = \begin{pmatrix} -56.1 & -1.6 & -13.6 & -6 \\ -115.2 & -2.5 & -27.6 & -12 \\ 233.1 & 8.8 & 57.7 & 24.3 \\ 53.1 & -3.2 & 10.2 & 7 \end{pmatrix}.$$

Is the origin an attractor, repeller or saddle point for the discrete dynamical system $x_{k+1} = Ax_k$? How do you know?

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & 4 & -4 & -4 \\ 2 & 9 & -12 & -9 \\ -4 & -18 & 25 & 19 \\ -1 & 0 & -15 & -2 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 1.1 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.6 & 0 \\ 0 & 0 & 0 & 1.9 \end{pmatrix}.$$

14. (1 point) Let

$$A = \begin{pmatrix} 12.3 & 2.4 & -1.1 \\ -30.8 & -5.8 & 2.8 \\ 59.4 & 12 & -5.2 \end{pmatrix}.$$

Is the origin an attractor, repeller or saddle point for the discrete dynamical system $x_{k+1} = Ax_k$? How do you know?

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & 2 & 1 \\ -4 & -7 & 0 \\ 2 & 6 & 11 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.2 \end{pmatrix}.$$

15. (1 point) Let

$$A = \begin{pmatrix} 28 & -6 & -42 \\ -22 & 4 & 34 \\ 26 & -6 & -40 \end{pmatrix}.$$

Is the origin an attractor, repeller or saddle point for the differential equation $y' = Ay$? How do you know?

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & -1 & -3 \\ -1 & 2 & 1 \\ 1 & -1 & -2 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} -8 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

16. (1 point) Let

$$A = \begin{pmatrix} -47 & 24 & 9 & -3 \\ 69 & -51 & -16 & 5 \\ -393 & 244 & 83 & -29 \\ -81 & 52 & 19 & -13 \end{pmatrix}.$$

Is the origin an attractor, repeller or saddle point for the differential equation $y' = Ay$? How do you know?

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & -2 & -1 \\ -2 & -7 & 11 & 4 \\ -3 & -7 & 3 & 4 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} -8 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -7 \end{pmatrix}.$$

17. (1 point) Let

$$A = \begin{pmatrix} 122 & -90 & -51 \\ 303 & -229 & -133 \\ -258 & 200 & 119 \end{pmatrix}.$$

Is the origin an attractor, repeller or saddle point for the differential equation $y' = Ay$? How do you know?

HINT: It may help to know that $AP = PD$ where

$$P = \begin{pmatrix} 1 & -3 & -3 \\ 3 & -8 & -5 \\ -3 & 7 & 2 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 6 \end{pmatrix}.$$

18. (1 point) Suppose $AP = PD$ where

$$P = \begin{pmatrix} 1 & -1 & -3 & 1 \\ 4 & 1 & 2 & 0 \\ 0 & 5 & 0 & -3 \\ 5 & -2 & -1 & -4 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} -9 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}.$$

Solve the discrete dynamical system $x_{k+1} = Ax_k$ where

$$x_0 = \begin{pmatrix} 8 \\ -19 \\ 10 \\ -46 \end{pmatrix}.$$

Show your work.

19. (1 point) Suppose $AP = PD$ where

$$P = \begin{pmatrix} 1 & -1 & -2 & 0 \\ 4 & 3 & -2 & 0 \\ 5 & -5 & 5 & -2 \\ -2 & 2 & -5 & -4 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} -4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Solve the initial value problem $y' = Ay$ where

$$y(0) = \begin{pmatrix} -4 \\ -32 \\ 12 \\ -6 \end{pmatrix}.$$

Show your work.

20. (1 point) Let

$$A = \begin{pmatrix} 7 & 24 & 16 \\ -16 & -43 & -26 \\ 16 & 39 & 22 \end{pmatrix} \quad \text{and} \quad x_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Use the Power Method to find estimates μ_5 and x_5 for the dominant eigenvalue of A and its eigenvector. Give your answer either as rational numbers or decimals with at least four digits of accuracy.

21. (1 point) Let

$$A = \begin{pmatrix} 3.7 & 5.2 & 3.6 \\ -31.2 & 12.1 & -10.4 \\ -21.6 & -10.4 & -14.3 \end{pmatrix}, \quad x_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and } \alpha = 2.$$

Use the Inverse Power Method to find the estimate ν_3 for the eigenvalue of A closest to α and the estimate x_3 for the corresponding eigenvector. Give your answer as decimals with at least four digits of accuracy.

Total for assignment: 21 points