# Section 2.1: Matrix Operations

September 18, 2017

# Outline

# From Last Time

Recall that there is a one-to-one correspondence between $m \times n$ matrices with entries in $\mathbb{R}$ and linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$.
Using this we can identify a linear transformation with its corresponding matrix and we'll denote a linear transformation by $L_A$ where $L_A(x) = Ax$ for the matrix $A$. Also recall that we can determine the columns of $A$ by

$$A_{*k} = L_A(I_{*k})$$

where $I$ is the $n \times n$ identity matrix.

From last time we know that if $A$ and $B$ are matrices then the function sum $L_A + L_B$ is defined iff $A$ and $B$ have the same size, and in this case $L_A + L_B$ is also a linear transformation.

**Definition.**

If $A$ and $B$ are two matrices having the same size then the *sum* $A + B$ is defined to be the matrix that satisfies the identity

$$L_{A+B} = L_A + L_B$$

# Addition: Definition and Calculation.

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**Observaton.**

$A_{m\times n} + B_{p\times q}$ is defined iff $m = p$ and $n = q$. In this case $A + B$ is also $m \times n = p \times q$.

# Multiplication: Definition and Calculation.

From last time we know that if $A$ and $B$ are matrices then the function compositon $L_A \circ L_B$ is defined iff the number of *columns* of $A$ equals the number of *rows* of $B$, and in this case $L_A \circ L_B$ is also a linear transformation.

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If $A$ and $B$ are two matrices with the number of columns of $A$ equaling the number of rows of $B$ then the *product* $AB$ is defined to be the matrix that satisfies the identity

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**Observation.**

$A_{m\times n}B_{p\times q}$ is defined iff $n = p$ and in this case $AB$ is $m \times q$.

# The Run Down.

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Because we defined addition and multiplication of matrices in terms of addition and composition of functions we get all the properties of these operations on functions for free. So, if the following products and sums are defined, we have:

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- $(AB)^T = B^T A^T$
- $(A + B)^T = A^T + B^T$