Section 2.1: Matrix Operations

September 18, 2017

Outline

Recall
Matrix Addition
Matrix Multiplication
Properties of Matrix Operations
Miscellaneous

From Last Time

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Recall that there is a one-to-one correspondence between $m \times n$ matrices with entries in $\mathbb{R}$ and linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$.

Using this we can identify a linear transformation with its corresponding matrix and we'll denote a linear transformation by L_A where $L_A(x) = Ax$ for the matrix A. Also recall that we can determine the columns of A by

$$A_{*k} = L_A(I_{*k})$$

where I is the $n \times n$ identity matrix.

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Definition and Calculation.

Addition: Definition and Calculation.

From last time we know that if A and B are matrices then the function sum $L_A + L_B$ is defined iff A and B have the same size, and in this case $L_A + L_B$ is also a linear transformation.

Definition.

If A and B are two matrices having the same size then the *sum* $A + B$ is defined to be the matrix that satisfies the identity

$$L_{A+B} = L_A + L_B$$

Calculation.

$$(A + B)_{*k} = (L_A + L_B)(I_{*k}) = L_A(I_{*k}) + L_B(I_{*k}) = A_{*k} + B_{*k}$$
$$(A + B)_{ij} = A_{ij} + B_{ij}$$

Observaton.

$A_{m \times n} + B_{p \times q}$ is defined iff $m = p$ and $n = q$. In this case $A + B$ is also $m \times n = p \times q$.

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Definition and Calculation.

Multiplication: Definition and Calculation.

From last time we know that if A and B are matrices then the function compositon $L_A \circ L_B$ is defined iff the number of *columns* of A equals the number of *rows* of B, and in this case $L_A \circ L_B$ is also a linear transformation.

Definition.

If A and B are two matrices with the number of columns of A equaling the number of rows of B then the *product* AB is defined to be the matrix that satisfies the identity

$$L_{AB} = L_A \circ L_B$$

Calculation.

$$(AB)_{*k} = (L_A \circ L_B)(I_{*k}) = L_A(L_B(I_{*k})) = A(B_{*k})$$
$$(AB)_{ij} \neq A_{ij}B_{ij}$$

Observation.

$A_{m \times n}B_{p \times q}$ is defined iff $n = p$ and in this case AB is $m \times q$.

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The Run Down.

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Things that work.

Because we defined addition and multiplication of matrices in terms of addition and composition of functions we get all the properties of these operations on functions for free. So, if the following products and sums are defined, we have:

(1) $(A + B) + C = A + (B + C)$

(2) $(AB)C = A(BC)$

(3) $(A + B)(C + D) = AC + BC + AD + BD$

(4) $0_{m \times n}$ (from the zero function $\mathbb{R}^n \to \mathbb{R}^m$)

(5) $I_{n \times n}$ (from the identity function $\mathbb{R}^n \to \mathbb{R}^n$)

(6) $A + B = B + A$ (only for addition)

Things that don't work.

Matrix multiplication has a few quirks:

- noncommutative
- zero divisors
- no cancellation

The matrices $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ are enough to illustrate these quirks.

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Odds and Ends.

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Powers.

$A^k = A \cdot A \cdots A$

Transpose.

The rows of A^T are the columns of A "laid down". The columns of A^T are the rows of A "stood up".

Properties of the Transpose

Compare with the properties of the inverse in Section 2.2.

- $(A^T)^T = A$
- $(AB)^T = B^T A^T$
- $(A + B)^T = A^T + B^T$