

Methods

We have two methods for finding the least squares solution to a (presumably inconsistent) linear system $Ax = b$. Here we assume A is $m \times n$ with $m \geq n$. Also, A is assumed to have linearly independent columns.

(Method I) Normal Equations

Operation		Flops
Compute	$A^T A$	mn^2
Compute	$A^T b$	mn
Solve	$(A^T A)x = A^T b$	n^3

(Method II) QDR Factorization

Operation		Flops
Factor	$A = QDR$	mn^2
Compute	$Q^T b$	mn
Solve	$Rx = Q^T b$	n^2

If both of n and $\kappa(A)$ are “relatively small”, then the methods are interchangeable. Otherwise the QDR factorization method is the preferred method, either for efficiency or accuracy reasons. Possibly both.

There is one caveat: I’ve ignored the coefficients in the *flops* estimates, and these can have an effect on the efficiency.