## Topology II HW 1 Due: Jan. 25

- (a) Suppose F is a continuous map from a compact space to a Hausdorff space. Prove that F is a homeomorphism if F is bijective.
  (b) Let R<sup>2\*</sup> be the compactification of R<sup>2</sup>. Give an example of open set in R<sup>2\*</sup> that is not in R<sup>2</sup>.
  (c) Prove that R<sup>2\*</sup> is homeomorphic to S<sup>2</sup>.
- 2. Let  $F: X \mapsto Y$  be a continuous and open map. Suppose X and Y are both compact and connected. Prove that F must be surjective.
- 3. Let K be a l-simplex in Euclidean space. Prove that  $\chi(K) = 1$ . Hint: Find the number of k simplex for  $k \leq l$  and use binomial Theorem.
- 4. Compute the Euler Characteristic of the Möbius band and the torus using Figure 5.7 on p105 and Figure 5.13 on page 115. (You only count distinct vertices, edges and faces).