Corrections to
Introduction to Topological Manifolds
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Changes or additions made in the past twelve months are dated.

- **Page 29, proof of Lemma 2.11:** Replace the first sentence of the proof by the following: "It suffices to show that $\mathcal{B}$ satisfies the two defining conditions for a basis, for then the fact that $\mathcal{B}$ consists of open sets guarantees that the topology generated by $\mathcal{B}$ is contained in the given topology on X, and conversely the hypothesis together with Lemma 2.10 implies that every open subset of X belongs to the topology generated by $\mathcal{B}$."
- **Page 29, paragraph before Exercise 2.15:** Instead of "the topologies of Exercise 2.1," it should say "some of the topologies of Exercise 2.1."
- **Page 30, last sentence of the proof of Lemma 2.12:** Replace U by $f^{-1}(U)$ (three times).
- **Page 30, first paragraph in the "Manifolds" section:** Delete the sentence "Let X be a topological space."
- **Page 38, Problem 2-16(b):** Replace part (b) by "Show that for any space Y, a map $f\colon X \to Y$ is continuous if and only if $p_n \to p$ in X implies $f(p_n) \to f(p)$ in Y."
- **Page 38, Problem 2-18:** This problem should be moved to Chapter 3, because $\operatorname{Int} M$ and ∂M are to be interpreted as having the subspace topologies. Also, for this problem, you may use without proof the fact that $\operatorname{Int} M$ and ∂M are disjoint.
- **Page 40, last line of Example 3.1:** Replace "subspace topology on B" by "subspace topology on C."
- **Page 45, line 15:** Change S^n to $\mathbb{S}^n$.
- **Page 47, line 5 from bottom:** Replace "next lemma" by "next theorem."
- **Page 51, proof of Proposition 3.13, third line:** $f_1(U_1), \dots, f_k(U_k)$ should be replaced by $f_1^{-1}(U_1), \dots, f_k^{-1}(U_k)$.
- **Page 51, proof of Proposition 3.14, last sentence:** Replace "the preceding lemma" by "the preceding proposition."
- **Page 52, first paragraph after Exercise 3.8:** In the first sentence, replace the words "surjective and continuous" by "surjective." Also, add the following sentence at the end of the paragraph: "It is immediate from the definition that every quotient map is continuous."
- **Page 52, last paragraph:** Change the word "quotient" to "surjective" in the first sentence of the paragraph.
- **Page 53, line 1:** Change the word "quotient" to "surjective" at the top of the page.
- **Page 53, Lemma 3.17:** Add the following sentence at the end of the statement of the lemma: (*More precisely, if $U \subset X$ is a saturated open or closed set, then $f|_U \colon U \to f(U)$ is a quotient map.*)
- **Page 57, second line after the first displayed diagram:** Replace the phrase "Y with the given topology is homeomorphic to Y with the quotient topology" with "the identity map is a homeomorphism between Y with the given topology and Y with the quotient topology."

(1/30/09) **Page 60, Example 3.35(a), line 4:** Change "by a linear transformation" to "by an invertible linear transformation."

- **Page 62, Problem 3-1:** The second part of the problem statement is false. Change the problem to the following: "Show that a finite product of open maps is open; give a counterexample to show that a finite product of closed maps need not be closed."
- **Page 62, Problem 3-4:** Add: "[Hint: For the unit ball in $\mathbb{R}^n$, consider the maps $\pi_i \circ \sigma^{-1}\colon \mathbb{R}^n \to \mathbb{R}^n$ for $1 \le i \le n$, where σ is stereographic projection and π_i is the projection from $\mathbb{R}^{n+1}$ to $\mathbb{R}^n$ that omits the ith coordinate.]"
- **Page 62, Problem 3-6:** Insert "nonempty" before "topological spaces."
- **Page 81, first displayed equation:** The definition of F should be

$$F(x) = \begin{cases} |x|\, f^{-1}\left(\dfrac{x}{|x|}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

- **Page 81, first line after the displayed equation:** Replace the first sentence by the following: "Then F is continuous away from the origin because f^{-1} is, and at the origin because boundedness of f^{-1} implies $F(x) \to 0$ as $x \to 0$."
- **Page 81, line 4:** Change $\mathbb{S}^n$ to $\mathbb{S}^{n-1}$.
- **Page 82, line 3 from bottom:** Delete "$= \overline{U} \cap Z$" from the sentence beginning "Since $\overline{U \cap Z}$ ...."
- **Page 83, Example 4.30(a):** In the first sentence, change "closed" to "open" and change $\overline{B}_\varepsilon(x)$ to $B_\varepsilon(x)$.
- **Page 85, statement of Corollary 4.34:** "*countable collection*" should read "*countable union*."
- **Page 89, Problem 4-11:** Insert "taking ∞ to ∞" after "$f^*\colon X^* \to Y^*$."
- **Page 94, Example 5.3, second line:** Change "Figure 5.3" to "Figure 5.4."
- **Page 96, Exercise 5.5:** Insert the words "isomorphic to" before "the vertex scheme."
- **Page 99, Lemma 5.4:** Replace part (d) by

  (*d*) *For any topological space Y, a map $F\colon |\mathcal{K}| \to Y$ is continuous if and only if its restriction to $|\sigma|$ is continuous for each $\sigma \in \mathcal{K}$.*

- **Page 102, line 13 from bottom:** Replace the two sentences beginning with "To prove this ..." by the following: "To prove this, suppose the contrary. Then there is some edge $e' \subset G_n$ such that either $\operatorname{Int} e \cap e'$ or $e \cap \operatorname{Int} e'$ is nonempty. Since $\operatorname{Int} e$ and $\operatorname{Int} e'$ are open subsets of M, it follows in either case that $\operatorname{Int} e \cap \operatorname{Int} e' \neq \varnothing$."
- **Page 103, Proposition 5.11:** In the statement of the proposition, change "*simplicial complex*" to "1-*dimensional simplicial complex*."
- **Page 105, line 2:** Replace 1977 by 1952 and [Moi77] by [Moi52].
- **Page 106, line 3 from bottom:** Replace "even" by "odd."
- **Page 111, Figure 5.12:** In $S(SK)$, the points inside the small triangles should be at the intersections of the three medians.

- **Page 114, Problem 5-2:** Replace the statement of the problem by: "Let $\mathcal{K}$ be an abstract simplicial complex. For each vertex v of $\mathcal{K}$, let $\operatorname{St} v$ (the *open star* of v) be the union of the open simplices $\operatorname{Int}|\sigma|$ as σ ranges over all simplices that have v as a vertex; and define a function $t_v\colon |\mathcal{K}| \to \mathbb{R}$ by letting $t_v(x)$ be the coefficient of v in the formal linear combination representing x.

  (a) Show that each function t_v is continuous.

  (b) Show that $\operatorname{St} v$ is a neighborhood of v, and the collection of open stars of all the vertices is an open cover of $|\mathcal{K}|$."

- **Page 114, Problem 5-3:** Delete the phrase "and locally path connected."

- **Page 114, Problem 5-5:** Insert the words "isomorphic to" before "the vertex scheme."

- **Page 120, Statement of Proposition 6.2(a):** Replace $x \in \partial\mathbb{B}^2$ by $(x, y) \in \partial\mathbb{B}^2$.

- **Page 125, line 2:** Insert the following sentence just before "This shows . . . ": "It is easy to check that $\widetilde{\alpha}$ is a bijective open map, and therefore a homeomorphism.

- **Page 126, Proposition 6.6:** Add the hypothesis that $n \geq 2$.

- **Page 127, second line from bottom:** Change $B_2(0)$ to $B_2(0) \smallsetminus \{0\}$.

- **Page 131, Part 1 of the definition of the geometric realization:** After "sides of length 1," insert "equal angles,".

- **Page 135, proof of Proposition 6.11:** Change S to M and S' to M' in the fifth line of the second paragraph of the proof, and again in the fifth and sixth lines of the third paragraph. [Here M and M' are supposed to denote the geometric realizations of various surface presentations.]

- **Page 135, middle of the page:** After "and whose restriction to each P_i is a homeomorphism," add the words "onto its image."

- **Page 136, line 8 from bottom:** Change the surface presentation in that line to $\langle S_1, S_2, a, b, c \mid W_1c^{-1}b^{-1}a^{-1}, abcW_2\rangle$.

(9/2/08) **Page 138, Theorem 6.14:** The first statement of the theorem should begin "*Any surface presentation of a connected surface is equivalent . . . .*" The last statement should begin "*Therefore, every nonempty connected compact surface . . . .*"

- **Page 139, proof of the classification theorem:** Replace the first sentence of the proof with "Let M be the compact surface determined by the given presentation."

- **Page 140, line 14:** Change "Step 3" to "Step 2."

- **Page 143, first line after the proof of Prop. 6.18:** Change "allows" to "allow."

- **Page 149, Example 7.3:** The first line should read "Define maps $f, g\colon \mathbb{R} \to \mathbb{R}^2$ by . . . ."

- **Page 156, Figure 7.7:** The labels $I \times I$, F, and X should all be in math italics.

- **Page 156, Exercise 7.2:** Change the first sentence to "Let X be a path connected topological space."

- **Page 159, second line from bottom:** "induced homeomorphism" should read "induced homomorphism."

- **Page 160, Proposition 7.18:** In the statement and proof of the proposition, change $(\iota_A)*$ to $(\iota_A)_*$ three times (the asterisk should be a subscript).

(8/22/09) **Page 166, line 10:** Change "space" to "group."

- **Page 174, proof of Lemma 7.35:** Change the word "maps" to "morphisms" (twice). Also, in the second-to-last line of the proof, change "Theorem 3.10" to "Theorem 3.11." (Actually, the last sentence is misleading, because the proof is not really exactly like that of Theorem 3.11. It would be clearer to replace the last sentence of the proof by the following: "If we take $W = P$ and $f_\alpha = \pi_\alpha$ in the diagram above, then the diagram commutes with either $f' \circ f$ or Id_P in place of f. By the uniqueness part of the defining property of the product, it follows that $f' \circ f = \mathrm{Id}_P$. A similar argument shows that $f \circ f' = \mathrm{Id}_{P'}$.")

- **Page 176, Problem 7-5:** Change "compact surface" to "connected compact surface."

- **Page 177, line 3:** The formula should read $\iota_\beta \colon X_\beta \hookrightarrow \coprod_\alpha X_\alpha$.

- **Page 188, proof of Theorem 8.7:** Replace the third sentence of the proof by "If $f : I \to \mathbb{S}^n$ is any loop based at a point in $U \cap V$, by the Lebesgue number lemma there is an integer m such that on each subinterval $[k/m, (k+1)/m]$, f takes its values either in U or in V. If $f(k/m) = N$ for some k, then the two subintervals $[(k-1)/m, k/m]$ and $[k/m, (k+1)/m]$ must be both be mapped into V. Thus, letting $0 = a_0 < \cdots < a_l = 1$ be the points of the form k/m for which $f(a_i) \neq N$, we obtain a sequence of curve segments $f|_{[a_{i-1}, a_i]}$ whose images lie either in U or in V, and for which $f(a_i) \neq N$." Also, in the last line of the proof, replace "f is homotopic to a path" by "f is path homotopic to a loop."

- **Page 189, proof of Proposition 8.9:** In the last sentence of the proof, change the domain of H to $I \times I$, and change the definition of H to

$$H(s,t) = (H_1(s,t), \ldots, H_n(s,t)).$$

- **Page 191, Problem 8-7:** In the third line of the problem, change $\varphi(\gamma)$ to $\varphi_*(\gamma)$.

- **Page 192, line 4:** Change the definition of φ to $\varphi(x) = (x - f(x))/|x - f(x)|$.

- **Page 199, second-to-last paragraph:** In the second sentence, after "a product of elements of S," insert "or their inverses."

(2/9/09) **Page 204, line 8 from the bottom:** After "coefficients are zero," insert the sentence "By convention, the empty set is considered to be independent."

(2/9/09) **Page 206, Example 9.15, first line:** Change "any finite group" to "any finite abelian group."

- **Page 208, Problem 9-4(b):** Change the first phrase to "Show that Ker $f_1 * f_2$ is equal to the normal closure of Im $j_1 * j_2, \ldots$." Add the following hint: "[Hint: Let N denote the normal closure of Im $j_1 * j_2$, so it suffices to show that $f_1 * f_2$ descends to an isomorphism from $(G_1 * G_2)/N$ to $H_1 * H_2$. Construct an inverse by showing that each composite map $G_j \hookrightarrow G_1 * G_2 \to (G_1 * G_2)/N$ passes to the quotient yielding a map $H_j \to (G_1 * G_2)/N$, and then invoking the characteristic property of the free product.]"

- **Page 213, proof of Proposition 10.5:** In the second sentence of the proof, change $\{q\}$ to $\{*\}$.

- **Page 218, Figure 10.4:** In the upper diagram, one of the arrows labeled a_i should be reversed.

- **Page 220, second line below the first displayed equation:** Change "clockwise" to "counterclockwise."

- **Page 227, line 8:** Replace $\overline{R} * \overline{S}$ by $\overline{R * S}$ (three times).

- **Page 233, last line:** Change the last sentence to "This brings us to the next-to-last major subject in the book: ...."

- **Page 238, proof of Proposition 11.10, second line:** Change "p maps ..." to "f maps ...."
- **Page 239, Example 11.14, last sentence:** Change $[b] \cdot [a]^{-1} \cdot [b]^{-1}$ to $[b] \cdot [a] \cdot [b]^{-1}$.
- **Page 247, Example 11.23:** Add the hypothesis that $n > 1$.
- **Page 248, Example 11.26:** Change $\mathcal{C}_\pi(\mathbb{P}^n)$ to $\mathcal{C}_\pi(\mathbb{S}^n)$.
- **Page 248, statement of Proposition 11.27(b):** Insert "(*with the discrete topology*)" after "*The covering group*".
- **Page 249, line 5:** Change the formula to "$p(\varphi(\widetilde{q})) = p(\widetilde{q}) = q$" (not p).
- **Page 253, Problem 11-9:** Change "path connected" to "locally path connected."
- **Page 265, Step 4:** In the second line of Step 4, replace "as in Step 3" by "as in Step 2."
- **Page 267, Proof of Prop. 12.9, second paragraph:** In the last sentence of the paragraph, replace $K \cap (g \cdot K) = \varnothing$ by $K \cap (g \cdot K) \neq \varnothing$.
- **Page 268, proof of Theorem 12.11:** The first and last paragraphs of this proof can be simplified considerably by using the result of Problem 3-15.
- **Page 272, first paragraph:** The last sentence should read "It can be identified with a quotient of the group of matrices of the form $\left(\begin{smallmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{smallmatrix}\right)$ with positive determinant (identifying two matrices if they differ by a scalar multiple), and so is a topological group acting continuously on $\mathbb{B}^2$."
- **Page 277, second paragraph from bottom:** After the sentence ending "when g and g' differ by a single edge transformation," insert the following: "An argument similar to that at the beginning of the proof shows that $g \cdot \widetilde{P}$ and $g' \cdot \widetilde{P}$ intersect in a vertex precisely when g and g' differ by a product of no more than $4n$ edge pairing transformations."
- **Page 277, line 6 from the bottom:** Change $(g\sigma^{-1}, \sigma(z_0))$ to $(g_0\sigma^{-1}, \sigma(z_0))$.
- **Page 281, lines 8 through 6 from the bottom:** Replace the sentence beginning "To prove this" by the following: "To prove this, let $K \subset \widetilde{M}$ denote the union of $\widetilde{P}$ together with its images $g \cdot \widetilde{P}$ under the finitely many $g \in G$ such that $\widetilde{P} \cap (g \cdot \widetilde{P}) \neq \varnothing$."
- **Page 283, proof of Corollary 12.18, second to last line:** Change "Corollary" to "Proposition."
- **Page 284, line 3:** Change "Since $\mathcal{C}_p(\widetilde{X})$ acts freely and properly on $\widetilde{X}$" to "Since $\mathcal{C}_p(\widetilde{X})$, endowed with the discrete topology, acts continuously, freely, and properly on $\widetilde{X}$".
- **Page 284, last displayed equation:** The last U on the right should be U'.
- **Page 287, line 10:** The sentence "Thus (i) corresponds to the rank 1 case" should read "Thus (ii) corresponds to the rank 1 case."
- **Page 289, Problem 12-5:** Replace the statement of the problem by "Find a group Γ acting freely and properly on the plane such that $\mathbb{R}^2/\Gamma$ is homeomorphic to the Klein bottle."
- **Page 290, Problem 12-9:** Replace the second sentence by "For any element $\widetilde{e}$ in the fiber over the identity element of G, show that $\widetilde{G}$ has a unique group structure such that $\widetilde{e}$ is the identity, $\widetilde{G}$ is a topological group, and the covering map $p\colon \widetilde{G} \to G$ is a homomorphism with discrete kernel."
- **Page 301, just above the third displayed equation:** In the last sentence of the paragraph, replace $G_{i,p}\colon \Delta_p \to \Delta_p \times I$ by $G_{i,p}\colon \Delta_{p+1} \to \Delta_p \times I$.

(3/4/09) **Page 301, just below the last displayed equation:** Change Δ to Δ_p (twice).

- **Page 316, first paragraph:** Change the fourth sentence to: "For $p > 0$, if $\alpha\colon \Delta_p \to \mathbb{R}^n$ is an affine p-simplex, set
$$s\alpha = \alpha(b_p) * s\partial\alpha$$
(where b_p is the barycenter of Δ_p), and extend linearly to affine chains."
- **Page 319, statement of Lemma 13.21:** H^{n-1} should be H_{n-1}.
- **Page 320, first paragraph:** In the last two lines, H^{n-1} should be H_{n-1} (twice).
- **Page 325, second to last displayed equation:** Change $H_p(\mathcal{K}'')$ to $H_p^{\Delta}(\mathcal{K}'')$.
- **Page 327, line 2:** Insert "retraction" after "strong deformation."
- **Page 330, paragraph after Exercise 13.4:** Replace [Mun75] by [Mun84].
- **Page 332, line 1:** The first word on the page should be "subgroups" instead of "spaces."
- **Page 333, line 7:** Change "coboundary" to "cocycle."
- **Page 334, Problem 13-8:** Replace [Mun75] by [Mun84].
- **Page 335, Problem 13-12:** Add the hypothesis that $U \cup V = X$.
- **Page 344, Exercise A.7(a):** Since this exercise requires the axiom of choice, it should be moved after exercise A.9.
- **Page 360, just before reference [Moi77]:** Insert the following reference:

  [Moi52] Edwin E. Moise. Affine structures in 3-manifolds. V. The triangulation theorem and Hauptvermutung. *Ann. of Math. (2)*, 56:96–114, 1952.