Topology I HW 3 Due: Sep. 21

1. Do problems 2-6, 2-8, 2-14, 2-16 on p36-p38.

2. Show that the only Hausdorff topology on a finite set is the discrete topology.

3. A set is said to have the finite complement topology if the closed sets are the finite sets together with the empty set. Let $f : R \mapsto R$ be the identity map $f(x) = x$ where in the domain R has the usual topology but in the codomain it has the finite complement topology. Show that f is continuous. Is f a homeomorphism? Explain your answer.

4. A set is said to have the countable-closed topology if the closed sets are the countable sets together with the empty set. If X is a topological space with an uncountably infinite number of points is the diagonal $\Delta := \{(x, x)|x \in X\}$ closed in the finite complement topology? Justify your answer carefully.

5. Prove that a space Y is Hausdorff if and only if for every topological space X and every pair of continuous functions $f : X \mapsto Y$ and $g : X \mapsto Y$, the set $\{x \in X|f(x) = g(x)\}$ is closed in X.