1. Let \((X, d)\) be a metric space and let \(A \subset X\). If \(x \in X\) define the distance of \(x\) to \(A\) to be \(\inf \{d(x, a) : a \in A\}\). Prove that the real-valued function on \(X\) defined by \(x \mapsto d(x, A)\) is continuous.

2. Prove the followings:
   (a) Show that every local homeomorphism is an open map.
   (b) Show that every homeomorphism is a local homeomorphism.
   (c) Show that a bijective continuous open map is a homeomorphism.
   (d) Show that a bijective local homeomorphism is a homeomorphism.

3. Prove the followings:
   Let \(X\) be a topological space and \(A \subset X\) any subset.
   (a) A point \(q \in \text{int } A\) iff \(q\) has a neighborhood contained in \(A\).
   (b) A point \(q \in \text{ext } A\) iff \(q\) has a neighborhood contained in \(X \setminus A\).
   (c) A point \(q \in \partial A\) iff every neighborhood of \(q\) contains both a point of \(A\) and a point of \(X \setminus A\).
   (d) \(\text{Int } A\) and \(\text{Ext } A\) are open in \(X\), while \(\overline{A}\) is closed in \(X\).
   (e) \(A\) is open iff \(A = \text{Int } A\),
   (f) \(A\) is closed iff it contains all its boundary points, which is true if and only if \(A = \text{Int } A \cup \overline{\partial A} = \overline{A}\).
   (g) \(\overline{A} = A \cup \partial A = \text{Int } A \cup \partial A\).
   (h) A set \(A\) in a topological space is closed iff it contains all of its limit points.

4. Prove that if \(B\) is a closed subset of \(X\) such that \(A \subset B\) then \(\overline{A} \subset B\).