Topology I HW 2 Due: Sep. 9

1. Let (X,d) be a metric space and let $A \subset X$. If $x \in X$ define the distance of x to A to be $\inf\{d(x,a) : a \in A\}$. Prove that the real-valued function on X defined by $x \mapsto d(x,A)$ is continuous.

2. Prove the followings:
   (a) Show that every local homeomorphism is an open map.
   (b) Show that every homeomorphism is a local homeomorphism.
   (c) Show that a bijective continuous open map is a homeomorphism.
   (d) Show that a bijective local homeomorphism is a homeomorphism.

3. Prove the followings:
   Let X be a topological space and $A \subset X$ any subset.

   (a) A point $q \in int\ A$ iff q has a neighborhood contained in A.

   (b) A point $q \in ext\ A$ iff q has a neighborhood contained in $X \backslash A$.

   (c) A point $q \in \partial\ A$ iff every neighborhood of q contains both a point of A and a point of $X \backslash A$.

   (d) $Int\ A$ and $Ext\ A$ are open in X , while $\overline{A}$ is closed in X.

   (e) A is open iff $A = IntA$,

   (f) A is closed iff it contains all its boundary points, which is true if and only if $A = IntA \cup \overline{A} = \overline{A}$.

   (g) $\overline{A} = A \cup \partial A = Int\ A \cup \partial A$.

   (h) A set A in a topological space is closed iff it contains all of its limit points.

4. Prove that if B is a closed subset of X such that $A \subset B$ then $\overline{A} \subset B$.

5. Do 2-2, 2-3 , 2-5 (on p36).