Chain rules

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Sum and Product rule: $(u \pm v)' = u' \pm v',\ (u \cdot v)' = u' \cdot v + u \cdot v'$

Quotient rule: $\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$

$$(u(x)^a)' = a \cdot u(x)^{a-1}(u(x))', \quad (e^{u(x)})' = e^{u(x)}(u(x))'$$

$$(\sin(u(x))' = \cos(u(x))u'(x)$$

$$(\cos(u(x))' = -\sin(u(x))u'(x)$$

$$(\tan(u(x))' = \sec^2(u(x))u'(x)$$

$$(\cot(u(x))' = -\csc^2(u(x))u'(x)$$

$$(\sec(u(x))' = \sec(u(x))\tan(u(x))u'(x)$$

$$(\csc(u(x))' = -\csc(u(x))\cot(u(x))u'(x)$$

- Find the derivatives of the following functions:
  e^{3x^2}, $\csc(3x^2)$, $\sin^{10}(x)$, $\sin^{10}(e^{x^2})$
  ,$4(\tan(x) - \sec(x))(tan(x) + \sec(x))$.

  Solution:
  1^0 $(e^{3x^2})' = e^{3x^2}(3x^2)'$ $(use(e^{u(x)})' = e^{u(x)}u'(x))$
  $= e^{3x^2} \cdot 6x = 6xe^{3x^2}$.

  2^0 $(\csc(3x^2))' = -\csc(3x^2)\cot(3x^2)(3x^2)'$
  $(use(\csc(u(x))' = -\csc(u(x))\cot(u(x))u'(x)$
  $= -\csc(3x^2)\cot(3x^2) \cdot 6x = -6x\csc(3x^2)\cot(3x^2)$.

  3^0 $(\sin^{10}(x))' = [(\sin(x))^{10}]'$ $(use\, \sin^{10}(x) = (\sin(x))^{10})$
  $= 10(\sin(x))^9 \cdot (\sin(x))'$
  $(use \quad (u(x)^{10})' = 10 \cdot u(x)^{10-1}u'(x)')$
  $= 10\sin^9(x) \cdot \cos(x)$.

$$
\begin{aligned}
&4^0\ (\sin^{10}(e^{x^2}))' = [(\sin(e^{x^2}))^{10}]' \\
&= 10(\sin(e^{x^2}))^9 \cdot (\sin(e^{x^2}))' \\
&= 10\sin^9(e^{x^2}) \cdot \cos(e^{x^2})(e^{x^2})'. \\
&= 10\sin^9(e^{x^2}) \cdot \cos(e^{x^2})e^{x^2}(x^2)' \\
&= 10\sin^9(e^{x^2}) \cdot \cos(e^{x^2})e^{x^2}(2x). \\
&= 20x\sin^9(e^{x^2}) \cdot \cos(e^{x^2})e^{x^2}.
\end{aligned}
$$

5^0 Find the derivative of $4(\tan(x) - \sec(x))(tan(x) + \sec(x))$

Note that
$4(\tan(x) - \sec(x))(tan(x) + \sec(x)) = 4(\tan^2(x) - \sec^2(x))$

$[4(\tan^2(x) - \sec^2(x))]' = 4(\tan^2(x) - \sec^2(x))' =$
$4([\tan^2(x)]' - [\sec^2(x)]')$

$= 4(2\tan(x)(\tan(x))' - 2\sec(x)(\sec(x))')$

$= 4(2\tan(x)\sec^2(x) - 2\sec(x)\sec(x)\tan(x))$

$= 4(2\tan(x)\sec^2(x) - 2\sec^2(x)\tan(x))$

$= 0$

Now Login MYMATHLAB
`http://portal.mypearson.com/cclogin.jsp` and work on tutorial for 3.5 and 3.6.