Differentiation rules

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Today's topics:

- Revisit tangent line problems
- Practice differentiation rules

Tangent rule

Goal: Use derivative to find the equation of the tangent line.

- It could be very complicated to find the slope of the tangent line by using the definition.

  Now we can use differentiation rules to find the the slope of the tangent line.

  Recall that the slope of the tangent line at $(a, f(a))$ is $f'(a)$.

  Now we know the point is $(a, f(a))$ and the slope is $f'(a)$.

  So the equation of the tangent line to $y = f(x)$ at $x = a$ is

  $y - f(a) = f'(a)(x - a)$.

Example

- Find the tangents to the graph of $y = \frac{3x}{-x^2+1}$ at the origin and the point $(2, -2)$ equation of the .

  First, let's find the derivative of $f(x) = \frac{3x}{-x^2+1}$.

  Using the quotient rule, we get
  $f'(x) = \frac{(3x)'(-x^2+1)-3x(-x^2+1)'}{(-x^2+1)^2}$

  $= \frac{3(-x^2+1)-3x(-2x)}{(-x^2+1)^2}$

  $= \frac{-3x^2+3+6x^2}{(-x^2+1)^2}$

  $= \frac{3x^2+3}{(-x^2+1)^2}$

  Now let's find the slope at the origin where $x = 0$.

  So $f'(0) = \frac{3 \cdot 0^2+3}{(-0^2+1)^2} = 3$. So the point is $(0, 0)$ and the slope is 3.

  So the tangent line equation at the origin is $y - 0 = 3(x - 0)$ and $y = 3x$.

Recall we have $f'(x) = \frac{3x^2+3}{(-x^2+1)^2}$

So the slope of the tangent line at $x = 2$ is

$f'(2) = \frac{3\cdot 2^2+3}{(-2^2+1)^2} = \frac{3\cdot 4+3}{(-4+1)^2} = \frac{12+3}{(-3)^2} = \frac{15}{9} = \frac{5}{3}$.

Now the point is $(2, -2)$ with slope $m = \frac{5}{3}$

So the tangent line equation at $(2, -2)$ is
$y - (-2) = \frac{5}{3}(x - 2) = \frac{5}{3}x - \frac{10}{3}$

and $y + 2 = \frac{5}{3}x - \frac{10}{3}$.

This gives $y = \frac{5}{3}x - \frac{10}{3} - 2 = \frac{5}{3}x - \frac{16}{3}$.

Constant rule:	$c' = 0$	$\frac{dc}{dx} = 0$
Power rule rule:	$(cx^a)' = c \cdot a \cdot x^{a-1}$	$\frac{d}{dx}(cx^a) = c \cdot a \cdot x^{a-1}$
Sum rule:	$(u \pm v)' = u' \pm v'$	$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
Product rule:	$(u \cdot v)' = u' \cdot v + u \cdot v'$	$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$
Quotient rule:	$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$	$\frac{d}{dx}(\frac{u}{v}) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Exponential ftn:	$(ce^{kx})' = c \cdot k \cdot e^{kx}$	$\frac{d}{dx}(ce^{kx}) = c \cdot k \cdot e^{kx}$

Practice differentiation rules

- Open the file by clicking `Z:/Math/Math1850/M4C13/MapletsForCalculus/MapletsForCalculus/files/maplets/DerivativeDrill.maplet`
- We will use this Maplet to practice differentiation rules.
- First, click on "Functions" to choose Polynomials and Exponentials .
- Now click on "Rules" to choose "Product Rules" and "Quotient rule".
- Now click on "Rules Per Problems" to choose "2".
- Now click on "Terms in Polynomial" to choose "3".
- Now click on "Power Properties" to choose "5" and "Allow negative".
- Now click on "New Derivative function".

- When you type your answer, you have to be careful about the bracket and the multiplication sign. $\frac{d}{dx}(3e^{-2x}(x^2+1))$.

  You should type
  $-6 * e\hat{}(-2 * x) * (x\hat{}2 + 1) + 3 * e\hat{}(-2 * x) * (2 * x)$ as your answer.

- Now do 10 problems.

- After you finish practicing the problem, take the quiz at MYMATHLAB
  `http://portal.mypearson.com/cclogin.jsp`.
  I will give you the password to take the quiz.