Rate of change and Inverse function

Mao-Pei Tsui

- Recall $f'(x)$ is the instantaneous rate of change of the function f.

- Suppose $s = f(t)$ is the position function of a moving object,

- The velocity function is $v(t) = f'(t)$,
  the speed function is $|v(t)| = |f'(t)|$
  the acceleration function is $a(t) = v'(t) = f''(t)$.

- In economics, if $c(x)$ is the cost function where x is the number of unit produced. Then $c'(x)$ is called the marginal cost of production which is the rate of change of cost with respect to level of production.

- A rock thrown vertically upward from the surface of the moon at a velocity of 16 m/sec reaches a height of $s(t) = 16t - 0.8t^2$ meters in t sec.
  (a) Find the rock's velocity and acceleration at time t.
  (b) How long does it take the rock to reach its highest point?
  (c) How high does the rock go?
  (d) How long does it take the rock to reach half its maximum height?
  (e)How long is the rock aloft?

  Solution: 1^0 The velocity is
  $v(t) = s'(t) = (16t - 0.8t^2)' = 16 - 1.6t$
  The acceleration is $a(t) = v'(t) = (16 - 1.6t)' = -1.6$

  2^0 The rock will reach its highest point when the velocity becomes zero, Solving $v(t) = 16 - 1.6t = 0$, we get
  $t = \frac{16}{1.6} = 10$.

3^0 (How high does the rock go?)
At $t = 10$, its height is $s(10) = 16 \cdot 10 - 0.8(10)^2 = 160 - 80 = 80$.
So the rock goes up to 80 meter.

4^0 (How long does it take the rock to reach half its maximum height?)
The maximum height is 80. So the half of its maximum height $= 80/2 = 40$.
To find the time it reach half its maximum height, we need to solve $s(t) = 40$, i.e $16t - 0.8t^2 = 40$.

This is the same as solving $-0.8t^2 + 16t - 40 = 0$. Using the quadratic formula, we get
$t = \frac{-16 \pm \sqrt{16^2 - 4 \cdot (-0.8)(-40)}}{2(-0.8)} = \frac{-16 \pm \sqrt{256 - 128}}{-1.6} = \frac{-16 \pm \sqrt{128}}{-1.6}$.So it reach half its maximum height when time is $\frac{-16 + \sqrt{128}}{-1.6}$ or $\frac{-16 - \sqrt{128}}{-1.6}$

5^0 (How long is the rock aloft?)
We need to know when the rock reach the ground. So we need to solve $s(t) = 0$.

Solving $16t - 0.8t^2 = 0$ or $t(16t - 0.8) = 0$, we get $t = 0$ or $t = \frac{16}{0.8} = 20$.

Hence the rock is aloft for 20 seconds.

Inverse functions.

- If a function is one to one, then we can define the inverse of f by $x = f^{-1}(y)$ if $y = f(x)$.
- the domain of f^{-1} = the range of f
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- The graph of $y = f^{-1}(x)$ and the graph $y = f(x)$ is symmetric with respect to the line $y = x$.
- To find the formula of $f^{-1}(x)$, we start from $y = f(x)$ and try to solve x in terms of y. Then we get $x = f^{-1}(y)$ and get a formula for $f^{-1}(y)$. Then we substitute y for x to get $f^{-1}(x)$.

- Find the formula of $f^{-1}(x)$ if $y = \frac{x-1}{2x+3}$.

  Solution: We want to express x in terms of y

  From $y = \frac{x-1}{2x+3}$, we have $y(2x+3) = x - 1 \Rightarrow$
  $2yx + 3y = x - 1$

  Now we try to solve for x in y.

  $\Rightarrow 2yx - x = -1 - 3y \Rightarrow x(2y-1) = -1 - 3y$

  $\Rightarrow x = \frac{-1-3y}{2y-1}$

  So $f^{-1}(y) = \frac{-1-3y}{2y-1}$ and $f^{-1}(x) = \frac{-1-3x}{2x-1}$.

Now work on "Recitation problems Oct 21 (grade will count)" in MYMATHLAB `http://portal.mypearson.com/cclogin.jsp`