Solution to Review Problems for Midterm II

Midterm II: Monday, October 18 in class
Topics: 3.1-3.7 (except 3.4)

1. Use the definition of derivative $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find the derivative of the functions.
(a) $f(x) = \sqrt{2x+3}$ (b) $f(x) = \frac{1}{2x+3}$.
Solution:
(a) First, let us find the expression
$f(x+h) - f(x) = \sqrt{2(x+h)+3} - \sqrt{2x+3} = \sqrt{2x+2h+3} - \sqrt{2x+3}$. Now
$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{2x+2h+3}-\sqrt{2x+3}}{h} = \frac{\sqrt{2x+2h+3}-\sqrt{2x+3}}{h} \cdot \frac{\sqrt{2x+2h+3}+\sqrt{2x+3}}{\sqrt{2x+2h+3}+\sqrt{2x+3}}$
$= \frac{(\sqrt{2x+2h+3})^2-(\sqrt{2x+3})^2}{h\cdot(\sqrt{2x+2h+3}+\sqrt{2x+3})} = \frac{2x+2h+3-(2x+3)}{h\cdot(\sqrt{2x+2h+3}+\sqrt{2x+3})} = \frac{2h}{h\cdot(\sqrt{2x+2h+3}+\sqrt{2x+3})} = \frac{2}{(\sqrt{2x+2h+3}+\sqrt{2x+3})}$.
Thus $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2}{\sqrt{2x+2h+3}+\sqrt{2x+3}}$
$= \frac{2}{\sqrt{2x+3}+\sqrt{2x+3}} = \frac{2}{2\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}}$.

(b) First, let us find the expression
$f(x+h) - f(x) = \frac{1}{2(x+h)+3} - \frac{1}{2x+3} = \frac{1}{2x+2h+3} - \frac{1}{2x+3}$
$= \frac{2x+3}{(2x+2h+3)(2x+3)} - \frac{2x+2h+3}{(2x+2h+3)(2x+3)} = \frac{2x+3-(2x+2h+3)}{(2x+2h+3)(2x+3)} = \frac{2x+3-2x-2h-3}{(2x+2h+3)(2x+3)} = \frac{-2h}{(2x+2h+3)(2x+3)}$.
Now $\frac{f(x+h)-f(x)}{h} = \frac{\frac{-2h)}{(2x+2h+3)(2x+3)}}{h} = \frac{-2h}{h(2x+2h+3)(2x+3)} = \frac{-2}{(2x+2h+3)(2x+3)}$. Thus
$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{-2}{(2x+2h+3)(2x+3)}$
$= \frac{-2}{(2x+3)(2x+3)} = \frac{-2}{(2x+3)^2}$.

2. Find the derivative of the following functions and simplify your answers.
(a) $12x^5 - \frac{3}{7x^2} + 4x^{\frac{-2}{5}}$ (b) $(1+4x)e^{-4x}$ (c) $(\sec(x)+\tan(x))^3$

(d) $x^5\cos(x) - 6x\sin(x) - 6\cos(x)$ (e) $(\frac{\cos(x)}{1+\sin(x)})^5$
Solution: (a) First, we rewrite $12x^5 - \frac{3}{7x^2} + 4x^{\frac{-2}{5}} = 12x^5 - \frac{3}{7}x^{-2} + 4x^{\frac{-2}{5}}$. So $(12x^5-\frac{3}{7x^2}+4x^{\frac{-2}{5}})' = (12x^5-\frac{3}{7}x^{-2}+4x^{\frac{-2}{5}})' = 12\cdot5\cdot x^4 - \frac{3}{7}\cdot(-2)x^{-3} + 4\cdot(\frac{-2}{5})\cdot x^{\frac{-7}{5}} = 60x^4 + \frac{6}{7}x^{-3} - \frac{8}{5}x^{\frac{-7}{5}}$.
(b) Applying the product rule, we have $((1+4x)e^{-4x})' = (1+4x)'e^{-4x} + (1+4x)(e^{-4x})' = 4e^{-4x} + (1+4x)e^{-4x}(-4x)' = 4e^{-4x} + (1+4x)e^{-4x}(-4) = 4e^{-4x} - 4e^{-4x} - 16xe^{-4x} = -16xe^{-4x}$.
(c) $[(\sec(x)+\tan(x))^3]' = 3(\sec(x)+\tan(x))^2(\sec(x)+\tan(x))'$
$= 3(\sec(x)+\tan(x))^2(\sec(x)\tan(x)+\sec^2(x))$
$= 3(\sec(x)+\tan(x))^2\cdot\sec(x)\cdot(\sec(x)+\sec(x)) = 3\sec(x)(\sec(x)+\tan(x))^3$.
(d)$(x^5\cos(x) - 6x\sin(x) - 6\cos(x))' = (x^5\cos(x))' - (6x\sin(x))' - (6\cos(x))' = 5x^4\cos(x) + x^5(-\sin(x)) - 6\sin(x) - 6x\cos(x) - 6(-\sin(x)) = 5x^4\cos(x) - x^5\sin(x) - 6\sin(x) - 6x\cos(x) + 6\sin(x) = 5x^4\cos(x) - x^5\sin(x) - 6x\cos(x)$.

(e) $[(\frac{\cos(x)}{1+\sin(x)})^5]' = 5(\frac{\cos(x)}{1+\sin(x)})^4(\frac{\cos(x)}{1+\sin(x)})' = 5(\frac{\cos(x)}{1+\sin(x)})^4(\frac{(\cos(x))'(1+\sin(x))-\cos(x)(1+\sin(x))'}{(1+\sin(x))^2})$
$= 5(\frac{\cos(x)}{1+\sin(x)})^4(\frac{(-\sin(x))(1+\sin(x))-\cos(x)\cos(x)}{(1+\sin(x))^2}) = 5(\frac{\cos(x)}{1+\sin(x)})^4(\frac{-\sin(x)-\sin^2(x)-\cos^2(x)}{(1+\sin(x))^2})$
$= 5(\frac{\cos(x)}{1+\sin(x)})^4(\frac{-\sin(x)-1}{(1+\sin(x))^2}) = 5(\frac{\cos(x)}{1+\sin(x)})^4(\frac{-1}{(1+\sin(x))}) = \frac{-5\cos^4(x)}{(1+\sin(x))^5}$.

3. Find the derivative of the following functions. You don't have to simplify your answer.

(a) $(2x+1)^3(1+e^{2x})^5$ (b) $\frac{(2x+1)^3}{(1+e^{2x})^5}$ (c) $\tan(\sin(xe^x))$ (d) $\cot^6(\frac{2}{t})$
(e) $\frac{7}{\sqrt[4]{x^2+e^{x^2}}}$ (f) $e^{\sec(x^2)}$ (g) $\sin^3(2t)\cos^3(2t)$ (h) $x^3\tan^3((1+x^2)^2)$
(i) $\frac{e^{x^2}\csc(3x)-x^2}{(1+x^2)^2}$ (j) $x^4e^{-3x}\cos(5x)$ (k) $\frac{\sin^{-5}(2x)}{x}-\frac{x\cos^3(2x)}{3}$ (l) $\sqrt{1+t\cos(t^2)-\frac{2t^3}{3}\sin(t^2)}$

Solution:(a) $[(2x+1)^3(1+e^{2x})^5]'$

$\underbrace{=}_{product\ rule} \underbrace{[(2x+1)^3]'}(1+e^{2x})^5 + (2x+1)^3\overbrace{[(1+e^{2x})^5]'}$

$= \underbrace{3(2x+1)^2\cdot(2x+1)'}(1+e^{2x})^5 + (2x+1)^3\cdot\overbrace{5\cdot(1+e^{2x})^4\cdot(1+e^{2x})'}$
$= 3(2x+1)^2\cdot 2\cdot(1+e^{2x})^5 + (2x+1)^3\cdot 5\cdot(1+e^{2x})^4\cdot e^{2x}\cdot(2x)'$
$= 6(2x+1)^2\cdot(1+e^{2x})^5 + (2x+1)^3\cdot 5\cdot(1+e^{2x})^4\cdot e^{2x}\cdot 2$
$= 6(2x+1)^2\cdot(1+e^{2x})^5 + 10(2x+1)^3(1+e^{2x})^4$

(b) First, we can rewrite $\frac{(2x+1)^3}{(1+e^{2x})^5} = (2x+1)^3(1+e^{2x})^{-5}$.
$[\frac{(2x+1)^3}{(1+e^{2x})^5}]' = [(2x+1)^3(1+e^{2x})^{-5}]'$

$\underbrace{=}_{product\ rule} \underbrace{[(2x+1)^3]'}(1+e^{2x})^{-5} + (2x+1)^3\overbrace{[(1+e^{2x})^{-5}]'}$

$= \underbrace{3(2x+1)^2\cdot(2x+1)'}(1+e^{2x})^{-5} + (2x+1)^3\cdot\overbrace{(-5)\cdot(1+e^{2x})^{-6}\cdot(1+e^{2x})'}$
$= 3(2x+1)^2\cdot 2\cdot(1+e^{2x})^{-5} + (2x+1)^3\cdot(-5)\cdot(1+e^{2x})^{-6}\cdot e^{2x}\cdot(2x)'$
$= 6(2x+1)^2\cdot(1+e^{2x})^{-5} + (2x+1)^3\cdot(-5)\cdot(1+e^{2x})^{-6}\cdot e^{2x}\cdot 2$
$= 6(2x+1)^2\cdot(1+e^{2x})^{-5} - 10(2x+1)^3(1+e^{2x})^{-6}$

(c) $[\tan(\sin(xe^x))]' = \sec^2(\sin(xe^x))[\sin(xe^x)]' = \sec^2(\sin(xe^x))\cos(xe^x)(xe^x)'$
$= \sec^2(\sin(xe^x))\cos(xe^x)(e^x+xe^x)$.

(d) Note that $\cot^6(\frac{2}{t}) = (\cot(2t^{-1}))^6$. So $[\cot^6(\frac{2}{t})]' = [(\cot(2t^{-1}))^6]'$
$= 6(\cot(2t^{-1}))^5\cdot[\cot(2t^{-1})]' = 6(\cot(2t^{-1}))^5\cdot[-\csc^2(2t^{-1})(2t^{-1})'$
$= 6(\cot(2t^{-1}))^5\cdot[-\csc^2(2t^{-1})](-2t^{-2})$.

(e) We can rewrite $\frac{7}{\sqrt[4]{x^2+e^{x^2}}} = \frac{7}{(x^2+e^{x^2})^{\frac{1}{4}}} = 7(x^2+e^{x^2})^{-\frac{1}{4}}$. So $(\frac{7}{\sqrt[4]{x^2+e^{x^2}}})' =$
$[7(x^2+e^{x^2})^{-\frac{1}{4}}]' = 7[(x^2+e^{x^2})^{-\frac{1}{4}}]'$

$= 7 \cdot (-\frac{1}{4}) \cdot (x^2 + e^{x^2})^{-\frac{5}{4}} \cdot (x^2 + e^{x^2})'$
$= 7 \cdot (-\frac{1}{4}) \cdot (x^2 + e^{x^2})^{-\frac{5}{4}} \cdot (2x + e^{x^2}(x^2)')$
$= 7 \cdot (-\frac{1}{4}) \cdot (x^2 + e^{x^2})^{-\frac{5}{4}} \cdot (2x + e^{x^2}(2x))$.
(f) $(e^{\sec(x^2)})' = e^{\sec(x^2)} \underbrace{(\sec(x^2))'} = e^{\sec(x^2)} \underbrace{\sec(x^2)\tan(x^2)(x^2)'}$
$= e^{\sec(x^2)} \sec(x^2)\tan(x^2) \cdot (2x)$.
(g) $(\sin^3(2t)\cos^3(2t))' = (\sin^3(2t))'\cos^3(2t) + \sin^3(2t)(\cos^3(2t))'$
$= 3(\sin^2(2t)) \cdot (\sin(2t))'\cos^3(2t) + \sin^3(2t) \cdot 3 \cdot (\cos^2(2t))(\cos(2t))'$
$= 3(\sin^2(2t)) \cdot \cos(2t) \cdot 2 \cdot \cos^3(2t) + \sin^3(2t) \cdot 3 \cdot (\cos^2(2t))(-\sin(2t)) \cdot 2$.
(h) $[x^3\tan^3((1+x^2)^2)]' = (x^3)'\tan^3((1+x^2)^2) + x^3[\tan^3((1+x^2)^2)]'$
$= 3x^2 \cdot \tan^3((1+x^2)^2) + x^3 \cdot 3\tan^2((1+x^2)^2)[\tan((1+x^2)^2)]'$
$= 3x^2 \cdot \tan^3((1+x^2)^2) + x^3 \cdot 3\tan^2((1+x^2)^2)[\sec^2((1+x^2)^2)][(1+x^2)^2]'$
$= 3x^2 \cdot \tan^3((1+x^2)^2) + x^3 \cdot 3\tan^2((1+x^2)^2)[\sec^2((1+x^2)^2)] \cdot 2 \cdot (1+x^2) \cdot (2x)$
(i)Note that $\frac{e^{x^2}\csc(3x)-x^2}{(1+x^2)^2} = (e^{x^2}\csc(3x) - x^2)(1+x^2)^{-2}$.
So $(\frac{e^{x^2}\csc(3x)-x^2}{(1+x^2)^2})' = [(e^{x^2}\csc(3x) - x^2)(1+x^2)^{-2}]'$
$= [(e^{x^2}\csc(3x) - x^2)]'(1+x^2)^{-2} + (e^{x^2}\csc(3x) - x^2)[(1+x^2)^{-2}]'$
$= ([e^{x^2}\csc(3x)]' - 2x)(1+x^2)^{-2} + (e^{x^2}\csc(3x) - x^2) \cdot (-2)[(1+x^2)^{-3}](1+x^2)'$
$= ((e^{x^2})'\csc(3x) + e^{x^2}(\csc(3x))' - 2x)(1+x^2)^{-2} + (e^{x^2}\csc(3x) - x^2) \cdot (-2)[(1+$
$x^2)^{-3}] \cdot (2x)$
$= ([e^{x^2} \cdot (2x) \cdot \csc(3x) + e^{x^2}(-\csc(3x)\cot(3x)) \cdot 3] - 2x)(1+x^2)^{-2}$
$+ (e^{x^2}\csc(3x) - x^2) \cdot (-2)[(1+x^2)^{-3}](2x)$.
(j) $(x^4e^{-3x}\cos(5x))' = (x^4e^{-3x})' \cdot \cos(5x) + x^4e^{-3x}(\cos(5x))'$
$= [(x^4)'e^{-3x} + x^4(e^{-3x})'] \cdot \cos(5x) + x^4e^{-3x}(-\sin(5x)) \cdot 5$
$= [4x^3 \cdot e^{-3x} + x^4 \cdot e^{-3x} \cdot (-3)] \cdot \cos(5x) + x^4e^{-3x}(-\sin(5x)) \cdot 5$.
(k) Note that $\frac{\sin^{-5}(2x)}{x} - \frac{x\cos^3(2x)}{3} = \sin^{-5}(2x)x^{-1} - \frac{1}{3}x\cos^3(2x)$.
So $(\frac{\sin^{-5}(2x)}{x} - \frac{x\cos^3(2x)}{3})' = (\sin^{-5}(2x)x^{-1})' - \frac{1}{3}(x\cos^3(2x))'$
$= (\sin^{-5}(2x))'x^{-1} + \sin^{-5}(2x)(x^{-1})' - \frac{1}{3}(x'\cos^3(2x) + x(\cos^3(2x))')$
$= (-5)\cdot(\sin^{-6}(2x))(\sin(2x))'x^{-1} + \sin^{-5}(2x)\cdot(-1)x^{-2} - \frac{1}{3}(\cos^3(2x) + x\cdot 3(\cos^2(2x))(\cos(2x))')$
$= (-5) \cdot (\sin^{-6}(2x))(\cos(2x)) \cdot 2 \cdot x^{-1} + \sin^{-5}(2x) \cdot (-1)x^{-2}$
$- \frac{1}{3}(\cos^3(2x) + x \cdot 3(\cos^2(2x))(-2\sin(2x)) \cdot 2)$
.
(l) Note that $\sqrt{1 + t\cos(t^2) - \frac{2t^3}{3}\sin(t^2)} = [1 + t\cos(t^2) - \frac{2t^3}{3}\sin(t^2)]^{\frac{1}{2}}$.
So $(\sqrt{1 + t\cos(t^2) - \frac{2t^3}{3}\sin(t^2)})' = ([1 + t\cos(t^2) - \frac{2t^3}{3}\sin(t^2)]^{\frac{1}{2}})'$
$= \frac{1}{2}[1 + t\cos(t^2) - \frac{2t^3}{3}\sin(t^2)]^{-\frac{1}{2}}[1 + t\cos(t^2) - \frac{2t^3}{3}\sin(t^2)]'$
$= \frac{1}{2}[1 + t\cos(t^2) - \frac{2t^3}{3}\sin(t^2)]^{-\frac{1}{2}}[t'\cos(t^2) + t(\cos(t^2))' - (\frac{2t^3}{3})'\sin(t^2) - \frac{2t^3}{3}(\sin(t^2))']$

$= \frac{1}{2}[1 + t\cos(t^2) - \frac{2t^3}{3}\sin(t^2)]^{-\frac{1}{2}}[\cos(t^2) + t(-\sin(t^2)) \cdot (2t) - (2t^2) \cdot \sin(t^2) - \frac{2t^3}{3}(\cos(t^2)) \cdot 2t]$.

4. Find the first derivative (y') and second derivative (y'') of the following functions.
(a) $y = (6 + \frac{4}{x})^5$ (b) $y = x^3 e^{3x}$

Solution: (a)Note that $y = (6 + \frac{4}{x})^5 = (6 + 4x^{-1})^5$. So $y' = [(6 + 4x^{-1})^5]' = 5(6 + 4x^{-1})^4(6 + 4x^{-1})' = 5(6 + 4x^{-1})^4(-4x^{-2}) = -20(6 + 4x^{-1})^4 x^{-2}$.
$y'' = [-20(6 + 4x^{-1})^4 x^{-2}]' = -20[(6 + 4x^{-1})^4]' x^{-2} - 20(6 + 4x^{-1})^4(x^{-2})'$
$= -20 \cdot 4 \cdot (6 + 4x^{-1})^3((6 + 4x^{-1}))' \cdot x^{-2} - 20(6 + 4x^{-1})^4 \cdot (-2x^{-3})$
$= -20 \cdot 4 \cdot (6 + 4x^{-1})^3 \cdot (-4x^{-2}) \cdot x^{-2} + 40(6 + 4x^{-1})^4 \cdot x^{-3}$
$= 320 \cdot (6 + 4x^{-1})^3 \cdot x^{-4} + 40(6 + 4x^{-1})^4 \cdot x^{-3}$.
(b) $y' = (x^3 e^{3x})' = (x^3)' e^{3x} + x^3 (e^{3x})' = 3x^2 e^{3x} + x^3 e^{3x} \cdot 3$
$= 3x^2 e^{3x} + 3x^3 e^{3x} = (3x^2 + 3x^3)e^{3x}$.
$y'' = [(3x^2 + 3x^3)e^{3x}]' = (3x^2 + 3x^3)' e^{3x} + (3x^2 + 3x^3)(e^{3x})'$
$= (6x + 9x^2)e^{3x} + (3x^2 + 3x^3)e^{3x} \cdot 3 = (6x + 9x^2)e^{3x} + (9x^2 + 9x^3)e^{3x}$
$= (6x + 9x^2 + 9x^2 + 9x^3)e^{3x} = (6x + 18x^2 + 9x^3)e^{3x}$.

5. Use implicit differentiation to find $\frac{dy}{dx}$.

(a) $2xy - y^2 = x$ (b) $x^3 + 3x^2 y + y^3 = 8$ (c) $\frac{x+y}{x-y} = x^2 + y^2$
(d) $\cos(xy) + x^5 = y^5$ (e) $e^{xy} = \sin(x + 5y)$

In implicit differentiation, Suppose $y = y(x)$ **then** $(f(y))' = f'(y)y'$
Solution:(a) Differentiating $2xy - y^2 = x$, we get $2(xy)' - (y^2)' = x' \Rightarrow 2x'y + 2xy' - 2yy' = 1 \Rightarrow 2y + 2xy' - 2yy' = 1$. Now we have $2xy' - 2yy' = 1 - 2y$
$\Rightarrow y'(2x - 2y) = 1 - 2y \Rightarrow y' = \frac{1-2y}{2x-2y}$.
(b) Differentiating the equation, we get $(x^3 + 3x^2 y + y^3)' = (8)'$
$\Rightarrow 3x^2 + 3(x^2)'y + 3x^2 y' + 3y^2 y' = 0$ and $3x^2 + 6xy + 2x^2 y' + 3y^2 y' = 0$. This implies that $2x^2 y' + 3y^2 y' = -3x^2 - 6xy$, $y'(2x^2 + 3y^2) = -3x^2 - 6xy$ and $y' = \frac{-3x^2 - 6xy}{2x^2 + 3y^2}$.
(c) Note that $\frac{x+y}{x-y} = x^2 + y^2$ is the same as $(x + y) = (x - y)(x^2 - y^2)$. Differentiating the equation, we get $(x + y)' = [(x - y)(x^2 - y^2)]'$
$\Rightarrow 1 + y' = (x - y)'(x^2 - y^2) + (x - y)(x^2 - y^2)' \Rightarrow$
$1 + y' = (1 - y')(x^2 - y^2) + (x - y)(2x - 2yy')$
$\Rightarrow 1 + y' = x^2 - y^2 - y'(x^2 - y^2) + 2x(x - y) - 2y(x - y)y'$
$\Rightarrow y' + y'(x^2 - y^2) + 2y(x - y)y' = x^2 - y^2 + 2x^2 - 2xy - 1 = 3x^2 - y^2 - 2xy - 1$
$\Rightarrow y'(1 + x^2 - y^2 + 2yx - 2y^2) = 3x^2 - y^2 - 2xy - 1$ and $y'(1 + x^2 - 3y^2 + 2yx) = 3x^2 - y^2 - 2xy - 1$. This gives $y' = \frac{3x^2 - y^2 - 2xy - 1}{1 + x^2 - 3y^2 + 2yx}$.

(d) $[\cos(xy) + x^5]' = (y^5)' \Rightarrow -\sin(xy)(xy)' + 5x^4 = 5y^4y'$
$-\sin(xy)(y + xy') + 5x^4 = 5y^4y' \Rightarrow -\sin(xy)y - \sin(xy)xy' + 5x^4 = 5y^4y'$
$-\sin(xy)xy' - 5y^4y' = \sin(xy)y - 5x^4 \Rightarrow y'(-\sin(xy)x - 5y^4) = \sin(xy)y - 5x^4$
$y' = \frac{\sin(xy)y-5x^4}{-\sin(xy)x-5y^4}$

(e) $(e^{xy})' = (\sin(x + 5y))' \Rightarrow e^{xy}(xy)' = \cos(x + 5y)(x + 5y)'$
$e^{xy}(y+xy') = \cos(x+5y)(1+5y') \Rightarrow e^{xy}y+e^{xy}xy' = \cos(x+5y)+5\cos(x+5y)y'$
$-5\cos(x + 5y)y' + e^{xy}xy' = \cos(x + 5y) - e^{xy}y$
$y'(-5\cos(x + 5y) + e^{xy}x) = \cos(x + 5y) - e^{xy}y$
$y' = \frac{\cos(x+5y)-e^{xy}y}{-5\cos(x+5y)+e^{xy}x}$.

6. Show that $(1, 2)$ lie on the curve $2x^3 + 2y^3 - 9xy = 0$. Then find the the tangent and normal to the curve at $(1, 2)$.
Solution: Plugging $(1, 2)$ to the equation $2x^3 + 2y^3 - 9xy$, we get $2 \cdot 1^3 + 2 \cdot 2^3 - 9 \cdot 1 \cdot 2 = 2 + 2 \cdot 8 - 18 = 2 + 16 - 18 = 0$. This means that $(1, 2)$ lie on the curve $2x^3 + 2y^3 - 9xy = 0$.
Next we find y' by implicit differentiation.
Differentiating $2x^3 + 2y^3 - 9xy = 0$, we get $2(x^3)' + 2(y^3)' - 9(xy)' = 0 \Rightarrow$ $6x^2 + 6y^2y' - 9y - 9xy' = 0 \Rightarrow 6y^2y' - 9xy' = -6x^2 + 9y$
$\Rightarrow y'(6y^2 - 9x) = -6x^2 + 9y \Rightarrow y' = \frac{-6x^2+9y}{6y^2-9x}$.

At $(1, 2)$, we have $y'(1) = \frac{-6\cdot 1^2+9\cdot 2}{6\cdot 2^2-9\cdot 1} = \frac{-6+18}{24-9} = \frac{12}{15} = \frac{4}{5}$. So the slope of the tangent line is $m = \frac{4}{5}$ and the point is $(1, 2)$. By the point slope formula, we have $y - 2 = \frac{4}{5}(x - 1)$.

From the slope of the tangent line, we know that the slope of the normal line is $m = -\frac{5}{4}$.
So the equation of the normal line is $y - 2 = -\frac{5}{4}(x - 1)$.

7. Find the normal to the curve $xy + 2x - y = 0$ that are parallel to the line $x + 2y = 0$.
Solution: First we find y' by implicit differentiation.
Differentiating $xy + 2x - y = 0$, we get $(xy)' + 2(x)' - (y)' = 0$
$\Rightarrow y + xy' + 2 - y' = 0 \Rightarrow xy' - y' = -y - 2$
$\Rightarrow y'(x - 1) = -y - 2 \Rightarrow y' = \frac{-y-2}{x-1}$. So the slope of the tangent line at (x, y) is $\frac{-y-2}{x-1}$. From here, we know that the slope of the normal line is $-\frac{x-1}{-y-2} = \frac{x-1}{y+2}$. The equation $x + 2y = 0$ can be rewritten as $2y = -x$ and $y = -\frac{1}{2}x$. So the slope is $m = -\frac{1}{2}$. At (x, y), the slope of the normal to the curve $xy + 2x - y = 0$ that are parallel to the line $x + 2y = 0$ must have slope $-\frac{1}{2}$. This implies that $\frac{x-1}{y+2} = -\frac{1}{2}$, $2x-2 = -y-2$ and $y = -2x$.

Plugging $y = -2x$ into the equation of the curve $xy + 2x - y = 0$, we get $x(-2x) + 2x - (-2x) = 0 \Rightarrow -2x^2 + 2x + 2x = 0 \Rightarrow -2x^2 + 4x = 0 \Rightarrow 2x(x-2) = 0 \Rightarrow x = 0$or $x = 2$. Recall that $y = -2x$. This implies that $y = 0$ or $y = -4$ So the point is $(0, 0)$ and the slope is $(2, -4)$. Recall the slope of the normal line is $-\frac{1}{2}$. So the normal line parallel to $x + 2y = 0$ are $y = -\frac{1}{2}x$ (point=$(0, 0)$)and $y + 4 = -\frac{1}{2}(x - 2)$ (point=$(2, -4)$).