16. a. $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} \blacksquare & * & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & * \end{bmatrix}$

In Exercises 17 and 18, determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

17. $\begin{bmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{bmatrix}$

18. $\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix}$

In Exercises 19 and 20, choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions. Give separate answers for each part.

19. $x_1 + hx_2 = 2$
$4x_1 + 8x_2 = k$

20. $x_1 + 3x_2 = 2$
$3x_1 + hx_2 = k$

In Exercises 21 and 22, mark each statement True or False. Justify each answer.[4]

21. a. In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.

b. The row reduction algorithm applies only to augmented matrices for a linear system.

c. A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.

d. Finding a parametric description of the solution set of a linear system is the same as *solving* the system.

e. If one row in an echelon form of an augmented matrix is $[\,0\ \ 0\ \ 0\ \ 5\ \ 0\,]$, then the associated linear system is inconsistent.

22. a. The echelon form of a matrix is unique.

b. The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.

c. Reducing a matrix to echelon form is called the *forward phase* of the row reduction process.

d. Whenever a system has free variables, the solution set contains many solutions.

e. A general solution of a system is an explicit description of all solutions of the system.

23. Suppose a 3×5 *coefficient* matrix for a system has three pivot columns. Is the system consistent? Why or why not?

24. Suppose a system of linear equations has a 3×5 *augmented* matrix whose fifth column is a pivot column. Is the system consistent? Why (or why not)?

25. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

26. Suppose the coefficient matrix of a linear system of three equations in three variables has a pivot in each column. Explain why the system has a unique solution.

27. Restate the last sentence in Theorem 2 using the concept of pivot columns: "If a linear system is consistent, then the solution is unique if and only if _____."

28. What would you have to know about the pivot columns in an augmented matrix in order to know that the linear system is consistent and has a unique solution?

29. A system of linear equations with fewer equations than unknowns is sometimes called an *underdetermined system*. Suppose that such a system happens to be consistent. Explain why there must be an infinite number of solutions.

30. Give an example of an inconsistent underdetermined system of two equations in three unknowns.

31. A system of linear equations with more equations than unknowns is sometimes called an *overdetermined system*. Can such a system be consistent? Illustrate your answer with a specific system of three equations in two unknowns.

32. Suppose an $n \times (n + 1)$ matrix is row reduced to reduced echelon form. Approximately what fraction of the total number of operations (flops) is involved in the backward phase of the reduction when $n = 30$? when $n = 300$?

Suppose experimental data are represented by a set of points in the plane. An **interpolating polynomial** for the data is a polynomial whose graph passes through every point. In scientific work, such a polynomial can be used, for example, to estimate values between the known data points. Another use is to create curves for graphical images on a computer screen. One method for finding an interpolating polynomial is to solve a system of linear equations.

(WEB)

---

[4]True/false questions of this type will appear in many sections. Methods for justifying your answers were described before Exercises 23 and 24 in Section 1.1.