**(Math 2890) Review Problems for Midterm II**

1. (a) What is a subspace in $R^n$?
(b) Is the set $\{(x,y,z)|x+y+z=1\}$ a subspace?
(c) Is the set $\{(x,y,z)|x-y-z=0, x+y-z=0\}$ a subspace?
(d) What is a basis for a subspace?
(e) What is the dimension of a subspace?
(f) What is the column space of a matrix?
(g) What is the null space of a matrix?
(h) What is an eigenvalue of a matrix $A$?
(i) What is an eigenvector of a matrix $A$?
(j) What is the characteristic polynomial of a matrix $A$ ?
(k) What is the subspace spanned by the vectors $v_1$, $v_2$, $\cdots$, $v_p$?

2. Find the inverses of the following matrices if they exist.

$$A = \begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

3. (a) Let $A$ be an $3\times 3$ matrix. Suppose $A^3+2A^2-3A+4I=0$. Is $A$ invertible? Express $A^{-1}$ in terms of $A$ if possible.
(b) Suppose $A^3=0$. Is $A$ invertible?

4. Describe the values of $t$ so that the following matrices are invertible

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 1 & t+1 & 3 \\ 1 & t & t+1 \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & 1 & 0 & t \\ -1 & 0 & t & 0 \\ 0 & -t & 0 & 1 \\ -t & 0 & -1 & 0 \end{bmatrix}$$

5. Find all values of $a$ and $b$ so that the subspace of $\mathbb{R}^4$ spanned by $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} b \\ 1 \\ -a \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ is two-dimensional.

6. Let $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \}$. You can assume that $\mathcal{B}$ is a basis for $R^3$

(a) Which vector $x$ has the coordinate vector $[x]_B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

(b) Find the $\beta$-coordinate vector of $y = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$.

7. Let

$$M = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 1 & 2 & 5 & 1 \\ 1 & 3 & 7 & 2 \end{bmatrix}$$

Find bases for $Col(M)$ and $Nul(M)$, and then state the dimensions of these subspaces

8. Find a basis for the subspace spanned by the following vectors $\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ -2 \end{bmatrix} \}$.

What is the dimension of the subspace?

9. Determine which sets in the following are bases for $\mathbb{R}^2$ or $\mathbb{R}^3$. Justify your answer

(a) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \end{bmatrix}$. (b) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$. (c) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(d) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. (e) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

10. Let $A$ be the matrix

$$A = \begin{bmatrix} -3 & -4 \\ -4 & 3 \end{bmatrix}.$$

(a) Find the characteristic polynomial of $A$.

(b) Find the eigenvalues and a basis of eigenvectors for $A$.

(c) Diagonalize the matrix $A$ if possible.

(d) Find a polynomial $P(A)$ in $A$ such that $P(A) = 0$. Verify your answer.

(e) Find the formula for $A^k$ where $k$ is an positive integer.

11. Let $A$ be the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

(a) Prove that $det(A - \lambda I) = -(\lambda - 1)^2(\lambda - 4)$.
(b) Find the eigenvalues and a basis of eigenvectors for A.
(c) Diagonalize the matrix A if possible.
(d) Find the matrix exponential $e^A$.

12. Let $B$ be the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find the characteristic equation of A.
(b) Find the eigenvalues and a basis of eigenvectors for A.
(c) Find a polynomial $P(A)$ in $A$ such that $P(A) = 0$. Verify your answer.
(d) Diagonalize the matrix A if possible.