Math 2890 Homework 7 Due date: Oct. 26

(1) Compute the determinant of the following matrices.

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 3 & 2 \\ -1 & 5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ -1 & -1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 0 & 4 \\ -1 & 5 & -3 & 2 \\ 2 & 3 & 0 & 2 \\ -1 & 5 & 0 & -1 \end{bmatrix}$$

(2) Find the characteristic polynomial, eigenvalues and eigenvectors of the following matrices. $\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$.

(3) (a) Let $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$.
Show that $det(A - \lambda I) = (1 - \lambda)(2 - \lambda)(3 - \lambda)$.
(b) Use the information above to find the eigenvalues and eigenvectors of A.
(c) Diagonalize the matrix A if possible, i.e find a invertible matrix P and a diagonal matrix where $A = PDP^{-1}$.

(4) (a) Let $A = \begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$.
Show that $det(A - \lambda I) = (1 - \lambda)(2 - \lambda)^2$.
(b) Use the information above to find the eigenvalues and eigenvectors of A.
(c) Diagonalize the matrix A if possible, i.e find a invertible matrix P and a diagonal matrix where $A = PDP^{-1}$.
(d) Find an expression for A^{10}.