Linear Algebra (Math 2890) Review Problems II

1. Find the inverses of the following matrices if they exist.

$$A = \begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}$$

2. Describe the values of t so that the following matrices are invertible

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 1 & t+1 & 3 \\ 1 & t & t+1 \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & 1 & 0 & t \\ -1 & 0 & t & 0 \\ 0 & -t & 0 & 1 \\ -t & 0 & -1 & 0 \end{bmatrix}$$

3. (a) Show that the matrix $\begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$ is invertible and find its inverse.

   (b) Use previous result to find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.

4. Find all values of a and b so that the subspace of $\mathbb{R}^4$ spanned by $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} b \\ 1 \\ -a \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ is two-dimensional.

5. Let $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \}$. You can assume that $\mathcal{B}$ is a basis for R^3

   (a) Which vector x has the coordinate vector $[x]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

   (b) Find the $\mathcal{B}$-coordinate vector of $y = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$.

6. Let

$$M = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 1 & 2 & 5 & 1 \\ 1 & 3 & 7 & 2 \end{bmatrix}$$

Find bases for $Col(M)$ and $Nul(M)$, and then state the dimensions of these subspaces

7. Determine which sets in the following are bases for $\mathbb{R}^2$ or $\mathbb{R}^3$. Justify your answer

(a) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \end{bmatrix}$. (b) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$.

(c) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (d) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(e) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

8. Diagonalize the following matrices if possible.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

9. Let A be the matrix

$$A = \begin{bmatrix} -3 & -4 \\ -4 & 3 \end{bmatrix}.$$

   (a) Find the eigenvalues and a basis of eigenvectors for A.
   (b) Diagonalize the matrix A if possible.
   (c) Find the matrix exponential e^A. Note that $e^A = \sum_{k=1}^{\infty} \frac{A^k}{k!} = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$.

10. Let $\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$.

(a) What is the column space of A?

(b) Describe the subspace $col(A)^{\perp}$ and find an basis for $col(A)^{\perp}$.

(c) Use Gram-Schmidt process to find an orthogonal basis for the column of the matrix A.

(d) Find an orthonormal basis for the column of the matrix A.

(e) Find the orthogonal projection of $y = \begin{bmatrix} -1 \\ 8 \\ -6 \\ 4 \end{bmatrix}$ onto the column space of A and write $y = \widehat{y} + z$ where $\widehat{y} \in col(A)$ and $z \in col(A)^{\perp}$. Also find the shortest distance from y to $Col(A)$.