Linear Algebra (Math 1890) Practice Problems 1

Midterm I: Feb 10, 2011, 12:30-1:45 at UH 1000 (Newton Lab).

Topics: 1.1, 1.2, 2.1,2.2, 2.3, 2.4.

1. Determine which of the following augmented matrices are in row echelon from, reduced row-echelon form or neither. Also determine which variables are free if it's in row echelon form or row reduced echelon form.
$$\begin{bmatrix} 2 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

2. Determine if the following systems are consistent and if so give all solutions in parametric vector form.
(a)
$$\begin{aligned} x_1 \quad -2x_2 \quad &= 3 \\ 2x_1 \quad -7x_2 \quad &= 0 \\ -5x_1 \quad +8x_2 \quad &= 5 \end{aligned}$$

(b)
$$\begin{aligned} x_1 \quad +2x_2 \quad -3x_3 \quad +x_4 \quad &= 1 \\ -x_1 \quad -2x_2 \quad +4x_3 \quad -x_4 \quad &= 6 \\ -2x_1 \quad -4x_2 \quad +7x_3 \quad -x_4 \quad &= 1 \end{aligned}$$

(c)
$$\begin{aligned} x_1 \quad +2x_2 \quad -3x_3 \quad +x_4 \quad &= 1 \\ -x_1 \quad -2x_2 \quad +4x_3 \quad -x_4 \quad &= 6 \\ -2x_1 \quad -4x_2 \quad +7x_3 \quad -2x_4 \quad &= 1 \end{aligned}$$

3. Let $A = \begin{bmatrix} 1 & 3 & -4 & 7 \\ 2 & 6 & 5 & 1 \\ 3 & 9 & 4 & 5 \end{bmatrix}$.

   (a) Find all the solutions of the non-homogeneous system $Ax = b$, and write them in parametric form, where $b = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$.

(b) Find all the solutions of the homogeneous system $Ax = 0$, and write them in parametric form.

4. Let $S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -4 \end{bmatrix} \right\}$.

(a) Is $v = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$ a linear combination of the vectors in S?

(c) Is $w = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$ a linear combination of the vectors in S?

5. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & b \\ -1 & 2 & a & 1 \end{array}\right]$$

(a) For what values of a will the system have a unique solution? What is the solution?(your answer may involve a and b)

(b) For what values of a and b will the system have infinitely many solutions?

(c) For what values of a and b will the system be inconsistent?

6. (a) Find the inverses of the following matrices if they exist.

$A = \begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}$.

(b) What's $(B^T)^{-1}$?

7. Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x + 2$. Show that $f(A) = 0_n$. Here 0_n is the $n \times n$ matrix.

8. Let A be an 3×3 matrix. Suppose $A^3 + 2A^2 - 4A + I_3 = 0$. Is A invertible? Express A^{-1} in terms of A if possible.

9. Express the following matrices as a product of elementary matrices and a matrix in reduced row-echelon form.
$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.